Fast Stability Analysis for Proportional-Integral Controller in Interval Systems

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(Received: 26-March-2018; accepted: 01-July-2018; published: 20-July-2018)
DOI: http://dx.doi.org/10.25073/jaec.201822.184

Abstract. *The paper describes a technique for stability analysis of proportional-integral (PI) controller in linear continuous-time interval control systems. The stability conditions of Kharitonov's theorem together with related criterions, such as Routh-Hurwitz criterion for continuous-time systems, bring out sets of polynomial inequalities. The sets are very difficult to solve directly, especially in case of high-order systems. Direct technique was used for stability analysis without solving polynomial inequalities. Solving polynomial equation directly makes its computing speed low. In the paper, a set theory-based technique is proposed for finding robust stability range of PI controller without solving any Kharitonov polynomials directly. Computation results confirm expected computing speed of the proposed technique.*

Keywords

Kharitonov polynomials, interval systems, proportional-integral (PI) controller, robust stability range, Routh-Hurwitz criterion.

1. Introduction

Stability analysis and design of controllers for multiple model or uncertain model systems require many complicated methods [1]-[4]. For linear continuous-time interval control systems which are linear continuous-time control systems with interval parameters, robust stability analysis is reduced by using Kharitonov's theorem [5]. The theorem provides an easy-implementing necessary and sufficient condition for Hurwitz stability of entire family of a set of polynomials - called interval polynomials [6]. In case of continuous-time systems, checking stability of the family is replaced by only checking stability of 4 or 8 polynomials in case of real-coefficient polynomials or complex-coefficient polynomials. In case of discrete-time systems, number of polynomials that must be checked for Hurwitz stability increases with system order and can be expressed as a sum of Euler functions [7].

Most of feedback controllers in the industrial processes are PI controllers [8]-[10] such as rotor speed adaptation mechanism of model reference adaptive system estimator in speed sensorless control of induction motor [11], parameter adaptions of induction motor [12, 13], fuzzy controller for intelligent gauge control system [14], pressure control of high pressure common rail injection system [15]. Finding robust stability ranges of the PI controller is necessary because of parameter uncertainty of the processes. This work consumes considerable time for solving polynomial inequalities received from Kharitonov's theorem and Routh-Hurwitz criterion [16].

Direct technique was used for solving the inequalities [17]. This technique is easy to understand, to compute, but it does not utilize the advantage of Routh-Hurwitz criterion: checking stability without solving characteristic polynomial directly. Therefore, its computing speed is low because of solving many polynomial equations, especially in case of high-order systems. Increasing computing speed is necessary for implementation into real control systems with digital signal processor. In this paper, a technique based on steps of checking stability using Routh-Hurtwitz criterion, is developed to overcome the disadvantage of direct technique. For implementation, an algorithm for solving polynomial inequality [18], intersections in set theory are described.

The remainder of the paper consists of 4 sections. Kharitonov's theorem and robust stability conditions are presented in the first section. Two techniques for finding stability range are described in next section. The third one presents computation examples. Conclusions and developments are carried out in the last one.

2. KHARITONOV'S THEOREM AND ROBUST STABILITY CONDITIONS

Kharitonov's theorem. A family $F(s)$ of interval real-coefficient polynomials of fixed order n is Hurwitz stable if and only if its four Kharitonov polynomials are Hurwitz stable [5, 6]. Form of $F(s)$ is:

$$F(s) = f_0 + f_1 s + f_2 s^2 + ... + f_n s^n \quad (1)$$

and its Kharitonov polynomials are:

$$K_1(s) = f_0^- + f_1^- s + f_2^+ s^2 + f_3^+ s^3 + ... \quad (2)$$

$$K_2(s) = f_0^- + f_1^+ s + f_2^+ s^2 + f_3^- s^3 + ... \quad (3)$$

$$K_3(s) = f_0^+ + f_1^- s + f_2^- s^2 + f_3^+ s^3 + ... \quad (4)$$

$$K_4(s) = f_0^+ + f_1^+ s + f_2^- s^2 + f_3^- s^3 + ... \quad (5)$$

where $f_i$coefficients, for $i = 0, 1, ..., n$, are bounded real numbers, and symbols $^-$, $^+$ respectively denote lower, upper bounders of coefficients. Next, consider the problem of checking robust stability of feedback linear continuous-time interval control system with a single input single output (SISO) plant $G(s)$, and a compensator or a controller $G_C(s)$ shown in Fig. 1. Family of interval transfer functions (FITF) of

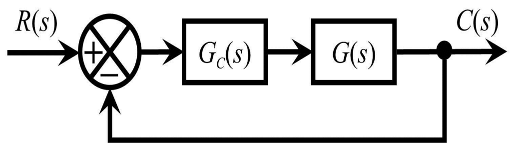

Fig. 1: Feedback linear continuous-time interval control systems.

the plant is given by:

$$G(s) = \frac{b_0 + b_1 s + b_2 s^2 + ... + b_n s^n}{a_0 + a_1 s + a_2 s^2 + ... + a_n s^n} \quad (6)$$

where degree n of denominator of $G(s)$ is guaranteed, and coefficients b_i, a_i for $i = 0, 1, 2, ..., n$ are limited in their given ranges:

$$a_i^- \leqslant a_i \leqslant a_i^+ \quad (7)$$

$$b_i^- \leqslant b_i \leqslant b_i^+ \quad (8)$$

For simplicity, the plant's FITF can be expressed as follow:

$$G(s) = \frac{[b_0^-, b_0^+] + [b_1^-, b_1^+]s + ... + [b_n^-, b_n^+]s^n}{[a_0^-, a_0^+] + [a_1^-, a_1^+]s + ... + [a_n^-, a_n^+]s^n} \quad (9)$$

And its Kharitonov transfer functions are given by:

$$G_1(s) = \frac{b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + ...}{a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + ...} \quad (10)$$

$$G_2(s) = \frac{b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + ...}{a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + ...} \quad (11)$$

Table 1. Description of used sets on Matlab software.

Set S	$\emptyset$	R	$(-\infty,\alpha)$	$(\alpha,+\infty)$	(α,β)
Description	[inf inf 0]	[inf inf 1]	$[\alpha\,\alpha\,2]$	$[\alpha\,\alpha\,3]$	$[\alpha\,\beta\,4]$

$$G_3(s)=\frac{b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + ...}{a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + ...} \quad (12)$$

$$G_4(s)=\frac{b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + ...}{a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + ...} \quad (13)$$

The system has family of interval characteristic equations as follow:

$$1+G_C(s)G(s)=0 \quad (14)$$

The compensator can be one of types: lead, lag, lead-lag, proportional (P), integral (I), derivative (D), PI, proportional-derivative (PD), proportional-integral-derivative (PID). In case of P, I, PI controllers, they can be described respectively by following transfer functions:

$$G_C(s)=k_P \quad (15)$$

$$G_C(s)=\frac{k_I}{s} \quad (16)$$

$$G_C(s)=k_P+\frac{k_I}{s} \quad (17)$$

Kharitonov polynomials are respectively given by Eqs. (18)-(21), (22)-(25), (26)-(29)

$$\begin{aligned}K_{1\_P}(s)&=(b_0^- k_P + a_0^-)+(b_1^- k_P + a_1^-)s\\&+(b_2^+ k_P + a_2^+)s^2+(b_3^+ k_P + a_3^+)s^3+...\end{aligned} \quad (18)$$

$$\begin{aligned}K_{2\_P}(s)&=(b_0^- k_P + a_0^-)+(b_1^+ k_P + a_1^+)s\\&+(b_2^+ k_P + a_2^+)s^2+(b_3^- k_P + a_3^-)s^3+...\end{aligned} \quad (19)$$

$$\begin{aligned}K_{3\_P}(s)&=(b_0^+ k_P + a_0^+)+(b_1^- k_P + a_1^-)s\\&+(b_2^- k_P + a_2^-)s^2+(b_3^+ k_P + a_3^+)s^3+...\end{aligned} \quad (20)$$

$$\begin{aligned}K_{4\_P}(s)&=(b_0^+ k_P + a_0^+)+(b_1^+ k_P + a_1^+)s\\&+(b_2^- k_P + a_2^-)s^2+(b_3^- k_P + a_3^-)s^3+...\end{aligned} \quad (21)$$

$$\begin{aligned}K_{1\_I}(s)&=(b_0^- k_I)+(b_1^- k_I + a_0^-)s\\&+(b_2^+ k_I + a_1^+)s^2+(b_3^+ k_I + a_2^+)s^3+...\end{aligned} \quad (22)$$

$$\begin{aligned}K_{2\_I}(s)&=(b_0^- k_I)+(b_1^+ k_I + a_0^+)s\\&+(b_2^+ k_I + a_1^+)s^2+(b_3^- k_I + a_2^-)s^3+...\end{aligned} \quad (23)$$

$$\begin{aligned}K_{3\_I}(s)&=(b_0^+ k_I)+(b_1^- k_I + a_0^-)s\\&+(b_2^- k_I + a_1^-)s^2+(b_3^+ k_I + a_2^+)s^3+...\end{aligned} \quad (24)$$

$$\begin{aligned}K_{4\_I}(s)&=(b_0^+ k_I)+(b_1^+ k_I + a_0^+)s\\&+(b_2^- k_I + a_1^-)s^2+(b_3^- k_I + a_2^-)s^3+...\end{aligned} \quad (25)$$

$$\begin{aligned}K_{1\_PI}(s)&=(b_0^- k_I)+(b_0^- k_P + b_1^- k_I + a_0^-)s\\&+(b_1^+ k_P + b_2^+ k_I + a_1^+)s^2\\&+(b_2^+ k_P + b_3^+ k_I + a_2^+)s^3+...\end{aligned} \quad (26)$$

$$\begin{aligned}K_{2\_PI}(s)&=(b_0^- k_I)+(b_0^+ k_P + b_1^+ k_I + a_0^+)s\\&+(b_1^+ k_P + b_2^+ k_I + a_1^+)s^2\\&+(b_2^- k_P + b_3^- k_I + a_2^-)s^3+...\end{aligned} \quad (27)$$

$$\begin{aligned}K_{3\_PI}(s)&=(b_0^+ k_I)+(b_0^- k_P + b_1^- k_I + a_0^-)s\\&+(b_1^- k_P + b_2^- k_I + a_1^-)s^2\\&+(b_2^+ k_P + b_3^+ k_I + a_2^+)s^3+...\end{aligned} \quad (28)$$

$$\begin{aligned}K_{4\_PI}(s)&=(b_0^+ k_I)+(b_0^+ k_P + b_1^+ k_I + a_0^+)s\\&+(b_1^- k_P + b_2^- k_I + a_1^-)s^2\\&+(b_2^- k_P + b_3^- k_I + a_2^-)s^3+...\end{aligned} \quad (29)$$

and constraints are respectively given by Eqs. (30), (31)-(32), (33)-(35):

$$b_i^- k_P + a_i^- \leqslant b_i^+ k_P + a_i^+, \forall i=0,1,2,...,n. \quad (30)$$

$$b_0^- k_I \leqslant b_0^+ k_I \quad (31)$$

$$b_i^- k_I + a_{i-1}^- \leqslant b_i^+ k_I + a_{i-1}^+, \forall i = 1, 2, ..., n. \tag{32}$$

$$b_0^- k_I \leqslant b_0^+ k_I \tag{33}$$

$$b_n^- k_P + a_n^- \leqslant b_n^+ k_P + a_n^+ \tag{34}$$

$$b_{i-1}^- k_P + b_i^- k_I + a_{i-1}^- \leqslant b_{i-1}^+ k_P + b_i^+ k_I + a_{i-1}^+, \tag{35}$$

$\forall i = 1, 2, ..., n$. The interval control system is robust stable if Kharitonov polynomials are Hurwitz stable. Routh-Hurwitz criterion was used to check Hurwitz stability of systems [19]-[22]. Necessary and sufficient conditions for stability is that all the terms of the first column of Routh array or all the determinants of the principal minors of Hurwitz matrix have the same sign [18]. In next sections, two techniques are used to find sets S_P, S_I of two parameters k_P, k_I that make the system stable.

3. TECHNIQUES FOR FINDING STABILITY RANGE

At first, two techniques are described for stability analysis in case of controllers with one parameter k_P or k_I. Assume that all roots of the terms of the first column of Routh array or the determinants of the principal minors of Hurwitz matrix were found. First one is the direct technique (DIT) that does not solve any inequalities which are generated from stability conditions. For each Kharitonov polynomial, processed steps of the technique are:

- Step 1: sort in ascending order distinct real roots of all the terms of the first column of Routh array or all the determinants of the principal minors of Hurwitz matrix: $k_1 < k_2 < ... < k_l$, where k is representative of k_P or k_I.
- Step 2: select the points $k = p_i$ for $i = 0, 1, 2, ..., l$ as follows:
  - interval $I_0 (k < k_1) : p_0 = 2k_1$, if $k_1 < 0$, or $p_0 = -1$, if $k_1 \geq 0$;
  - interval $I_i (k_i < k < k_{i+1}) : p_i = (k_i + k_{i+1})/2$, for $i = 1, 2, ..., l-1$;
  - interval $I_l (k > k_l) : p_l = 2k_l$, if $k_l > 0$ or $p_l = 1$, if $k_l \leqslant 0$.
- Step 3: for each value $k = p_i$, find all roots of each Kharitonov polynomial. If all roots have negative real parts, interval I_i satisfies stability condition.

The second technique is the set theory-based on technique (SBT) that solves the polynomial inequalities, and uses basic intersection in set theory. Description of used sets on Matlab software is shown in Tab. 1. Intersection of two sets is implemented according to basic rules of set theory (see Tab. 2). Two characters m, M denote min, max functions respectively. For each Kharitonov polynomial, it is described by following steps:

- Step 1: assume that each term of the first column of Routh array or each determinant of the principal minors of Hurwitz matrix, is a r^{th}-order polynomial $P(k)$, and coefficient c_r associates with k^r ($c_r \neq 0$). Sort its distinct odd-multiplicity real roots in ascending order: $k_1 < k_2 < ... < k_q$ ($q \leq r$).
- Step 2: no loss of generality, solve the inequality $P(k) > 0$ by an algorithm shown in Fig. 3.
- Step 3: apply intersection to find range of k which satisfies all inequalities.

Two described techniques are applied for all Kharitonov polynomials. The intersection is used to obtain the set S_P or the set S_I that satisfies the stability conditions. In case of PI controller, at first, the initial value V_I, the final value V_F, the value of increment Δ_V of parameter k_P or k_I are given. Then, for each value of k_P or k_I, the set S_I or S_P is found by checking stability of 4 Kharitonov polynomials (see Eqs. (26)-(29)). The intersections of these sets S_I or S_P are the final results.

Table 2. Intersection of two sets.

Set S_2	Set S_1: $(-\infty,\alpha_1)$	Set S_1: $(\alpha_1,+\infty)$	Set S_1: (α_1,β_1)
$(-\infty,\alpha_2)$	$(-\infty,m(\alpha_1,\alpha_2))$	$\begin{cases}(\alpha_1,\alpha_2) \text{ if } \alpha_1<\alpha_2\\ \emptyset, \text{ otherwise}\end{cases}$	$\begin{cases}(\alpha_1,m(\alpha_2,\beta_1)) \text{ if } \alpha_1<\alpha_2\\ \emptyset, \text{ otherwise}\end{cases}$
$(\alpha_2,+\infty)$	$\begin{cases}(\alpha_2,\alpha_1) \text{ if } \alpha_2<\alpha_1\\ \emptyset, \text{ otherwise}\end{cases}$	$(M(\alpha_1,\alpha_2),+\infty)$	$\begin{cases}(M(\alpha_1,\alpha_2),\beta_1) \text{ if } \alpha_2<\beta_1\\ \emptyset, \text{ otherwise}\end{cases}$
(α_2,β_2)	$\begin{cases}(\alpha_2,m(\alpha_1,\beta_2),\beta_1)\\ \text{if } \alpha_2<\alpha_1\\ \emptyset, \text{ otherwise}\end{cases}$	$\begin{cases}(M(\alpha_1,\alpha_2),\beta_2)\\ \text{if } \alpha_1<\beta_2\\ \emptyset, \text{ otherwise}\end{cases}$	$\begin{cases}\emptyset, \text{ if } \alpha_2\ge\beta_1\\ \text{or } \alpha_1\ge\beta_2\\ (M(\alpha_1,\alpha_2),m(\beta_1,\beta_2)), \text{ otherwise}\end{cases}$

Table 3. Selected plants.

n	$G(s)$
2	$\frac{[36,44]+[4.3,5.7]s}{[54,66]+[5.7,8.3]s+[1,1]s^2}$
3	$\frac{[3.2,4.3]+[2.3,3.7]s+[1.1,1.9]s^2}{[11.7,14.9]+[7.5,9.6]s+[3.3,5.2]s^2+[1,1]s^3}$
4	$\frac{[7.5,12.5]+[17,23]s+[12,18]s^2+[3.5,6.5]s^3}{[10.5,17.5]+[23,37]s+[15,25]s^2+[3,7]s^3+[1,1]s^4}$
5	$\frac{[46,54]+[85,125]s+[90,110]s^2+[27,34]s^3+[4,6]s^4}{[63,77]+[150,198]s+[115,135]s^2+[52,58]s^3+[8,10]s^4+[1,1]s^5}$
6	$\frac{[320,380]+[554,574]s+[950,1050]s^2+[225,245]s^3+[90,110]s^4+[10,12]s^5}{[340,400]+[1150,1250]s+[604,644]s^2+[470,530]s^3+[70,80]s^4+[9,11]s^5+[1,1]s^6}$
7	$\frac{[329,471]+[706,865]s+[558,643]s^2+[282,319]s^3+[70,83]s^4+[12,15]s^5+[1.0,1.4]s^6}{[387,521]+[877,1024]s+[711,889]s^2+[326,360]s^3+[89,110]s^4+[13.3,16.7]s^5+[1.2,1.6]s^6+[0.1,0.1]s^7}$

Table 4. Sets S_P, S_I of P and I controllers.

n	S_P (P controller)	S_I (I controller)
2	$(-1.325581395348837,+\infty)$	$(0,15.792714212416620)$
3	$(-1.136952577372862,+\infty)$	$(0,10.395928891361976)$
4	$(-0.071101889488303,+\infty)$	$(0,0.373239166328192)$
5	$(-0.857142857142857,+\infty)$	$(0,49.784749592528634)$
6	$(-0.020328133920827,+\infty)$	$(0,0.767612236055811)$
7	$(-0.079686910635607,+\infty)$	$(0,1.646059600788306)$

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4. COMPUTATION EXAMPLES

Two techniques are implemented on Matlab software R2014a, version 8.3.0.532 with processor Intel Core i7-6700HQ CPU 2.6GHz, installed memory (RAM) 8.00 GB (7.88 GB usable). Hurwitz matrix is used to avoid the error due to polynomial division in calculations of Routh array. All FITFs of selected plants that listed in Tab. 3 have relative degree of 1. For P, I controllers, sets S_P, S_I are calculated and listed in Tab. 4. Because boundaries α_I, β_I of all sets S_I are limited (see Tab. 4), so values $\Delta_V, \Delta_I, \Delta_F$ of parameter k_I are chosen as follows:

$$\Delta_V = \frac{\beta_I - \alpha_I}{101} \tag{36}$$

$$V_I = \alpha_I + \Delta_V \tag{37}$$

$$V_F = \beta_I - \Delta_V \tag{38}$$

Computing time (CT) is the time that the processor executes all steps for 4 Kharitonov polynomials with 100 given values of k_I (see Eqs. (36) – (38)). For comparison of two techniques, two functions *tic*, *toc* are used to measure their CT. Statistically, two techniques are run 30 times, and minimum, maximum, average values of CT ($CT_{min}, CT_{max}, CT_{avg}$) are listed in Tab. 5. The CTs of SBT are much smaller than those of DIT. Ratios of CTs can be defined as follows:

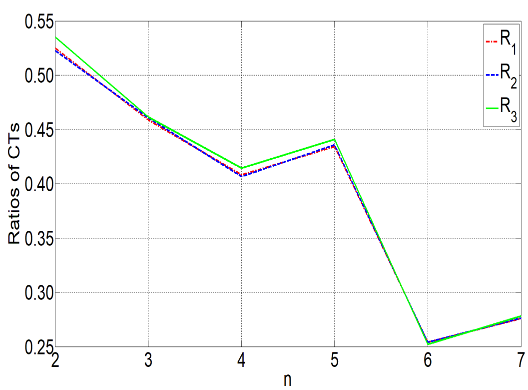

Fig. 2: Ratios of CTs.

$$R_1 = \frac{CT_{\min} \text{ of SBT}}{CT_{\min} \text{ of DIT}} \tag{39}$$

$$R_2 = \frac{CT_{avg} \text{ of SBT}}{CT_{avg} \text{ of DIT}} \tag{40}$$

$$R_3 = \frac{CT_{\max} \text{ of SBT}}{CT_{\max} \text{of DIT}} \tag{41}$$

Figure 2 shows these ratios that are smaller than one in all situations. They tend to decrease with the increase of n, exceptionally for changes of n from 4 to 5 and from 6 to 7.

For the DIT, in most cases, the higher degree n, the longer CTs, except for values $n = 6, 7$. For each Kharitonov polynomial, the step 3 of this technique is performed $(l + 1)$ times where parameter l is number of distinct real roots of all the determinants of the principal minors of Hurwitz matrix. For DIT, parameter l, number of Kharitonov polynomials n_l with the same parameter l, and number of times that step 3 is performed n_{s3}, are listed in Tab. 6. It can easy to see that the parameter which affects CTs of DIT most is n_{s3}. Especially, n changes from 6 to 7, n_{s3} decrease from 8149 to 7209, this makes CTs shorter. In cases of $n = 4, 5$, the values of n_{s3} are equivalent, therefore CTs increase insignificantly.

For SBT, order q of inequality, number of q^{th}-degree inequalities n_q, number of times that step 2 is performed (n_{s2}), number of intersections n_i are listed in Tab. 7. It is easy to see that $n_{s2} = 400(n + 1)$. The higher the n_{s2} is, the longer the CTs are. Besides that, CTs is significantly dependent on the parameter n_i. This value of n_i (6956) for $n = 6$ is larger than that (6207) for $n = 7$. This increment makes CTs increase insignificantly although for $n = 7$, n_{s2} (3200) is larger than that (2800) for $n = 6$.

Table 5. Values $CT_{min}, CT_{max}, CT_{avg}$ [ms].

n	CT_{min}		CT_{avg}		CT_{max}		Sp (PI controller)
	DIT	SBT	DIT	SBT	DIT	SBT	
2	57.9	30.4	60.1	31.4	64.1	34.3	$(-1.325581395348837, +\infty)$
3	76.5	35.1	78.8	36.3	82.1	37.9	$(-1.136952577372862, +\infty)$
4	124.0	50.6	125.4	51.0	127.6	52.9	$(-0.071101889488303, +\infty)$
5	134.5	58.4	137.0	59.7	146.8	64.7	$(-0.857142857142857, +\infty)$
6	343.1	87.2	346.6	88.1	352.8	89.0	$(-0.020328133920827, +\infty)$
7	329.2	90.8	332.1	91.8	342.0	95.1	$(-0.079686910635607, +\infty)$

Table 7(a)-Parameters of SBT.

n											
2				3				4			
q	n_q	n_{s2}	n_i	q	n_q	n_{s2}	n_i	q	n_q	n_{s2}	n_i
1	400	1200	2000	1	1178	1600	2022	1	659	2000	3399
2	800			2	400			2	1142		
				3	22			3	141		
								4	58		

Table 7(b)-Parameters of SBT.

n											
5				6				7			
q	n_q	n_{s2}	n_i	q	n_q	n_{s2}	n_i	q	n_q	n_{s2}	n_i
0	11	2400	3207	1	400	2800	6956	1	699	3200	6217
1	1596			2	400			2	1036		
2	789			3	400			3	941		
3	4			4	1044			4	148		
				5	400			5	330		
				6	156			6	16		
								7	30		

Table 6. Parameters of DIT $(n = \overline{2,7}\ )$.

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<tr><td>n_l</td><td colspan="20">400</td></tr>
<tr><td>n_{s3}</td><td colspan="20">1600</td></tr>
<tr><td rowspan="3">3</td><td>l</td><td colspan="10">4</td><td colspan="10">6</td></tr>
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<tr><td>n_{s3}</td><td colspan="20">2022</td></tr>
<tr><td rowspan="3">4</td><td>l</td><td colspan="7">6</td><td colspan="7">8</td><td colspan="6">10</td></tr>
<tr><td>n_l</td><td colspan="7">259</td><td colspan="7">112</td><td colspan="6">29</td></tr>
<tr><td>n_{s3}</td><td colspan="20">3140</td></tr>
<tr><td rowspan="3">5</td><td>l</td><td colspan="7">5</td><td colspan="7">7</td><td colspan="6">9</td></tr>
<tr><td>n_l</td><td colspan="7">10</td><td colspan="7">387</td><td colspan="6">3</td></tr>
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<tr><td>n_l</td><td colspan="7">322</td><td colspan="7">7</td><td colspan="6">71</td></tr>
<tr><td>n_{s3}</td><td colspan="20">8149</td></tr>
<tr><td rowspan="3">7</td><td>l</td><td>14</td><td>15</td><td>16</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td><td>31</td><td>32</td><td>33</td><td>34</td></tr>
<tr><td>n_l</td><td>248</td><td>1</td><td>19</td><td>24</td><td>1</td><td>26</td><td>5</td><td>18</td><td>6</td><td>17</td><td>3</td><td>4</td><td>11</td><td>5</td><td>2</td><td>1</td><td>1</td><td>1</td><td>5</td><td>2</td></tr>
<tr><td>n_{s3}</td><td colspan="20">7209</td></tr>
</table>

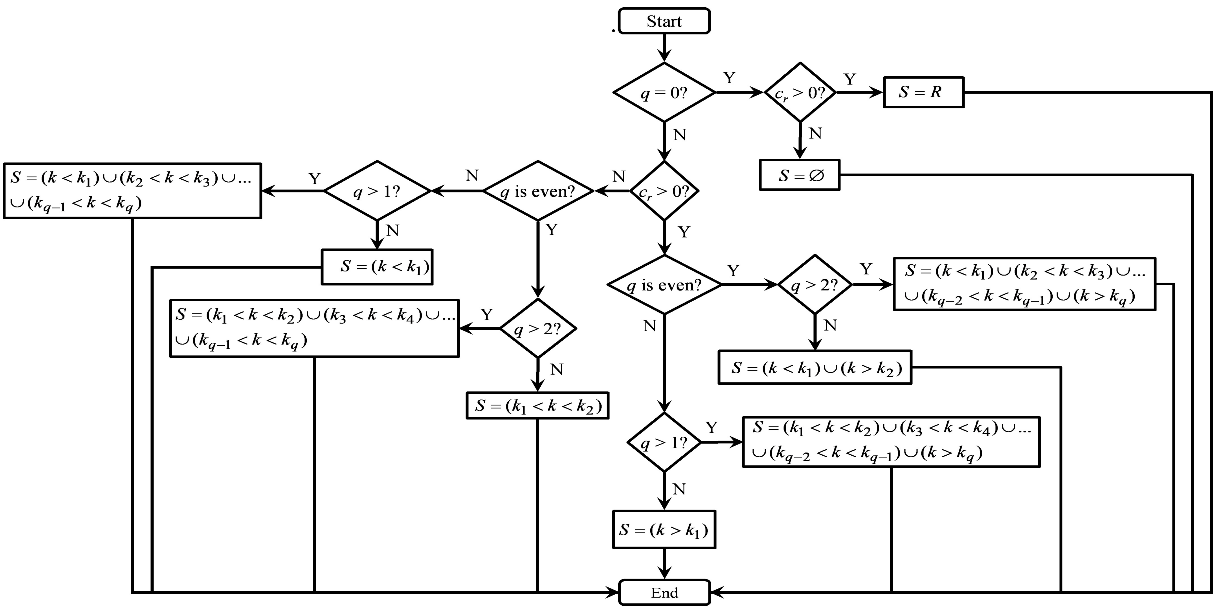

Fig. 3: Algorithm for solving the polynomial inequality $P(k) > 0$.

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5. CONCLUSIONS

Two techniques was developed to find stability range of proportional-integral controller for linear continuous-time interval control systems. Set-based theory technique uses the advantage of stability criterions: checking stability without solving any Kharitonov polynomials directly. It gives computing time much shorter than direct technique does, especially with high-order systems. Therefore, it can be applied to obtain boundaries of PI-based or PID-based intelligent controllers for real systems [3, 23]. Combination with high-accuracy system order reduction methods can decrease computing time [24]. Stability analysis and design of controllers for fractional-order systems can be done similarly to the works for systems with rational-order transfer functions by approximating the systems using real interpolation method (RIM) with high-order models [25]. The main drawback of this method that is the uncertainty of approximation model is overcome by Kharitonov's theorem. This computing technique can be extended for finding stability range of feedback linear discrete-time interval control systems [7], nonlinear systems with time-varying delay [4].

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