# Decentralized Controller Design for Large Scale Switched Takagi-Sugeno Systems with H $\infty$ Performance Specifications 

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(Received: 09-May-2018; accepted: 21-June-2018; published: 20-July-2018)
DOI: http://dx.doi.org/10.25073/jaec.201822.187


#### Abstract

This paper investigates the design of decentralized controllers for a class of large scale switched nonlinear systems under arbitrary switching laws. A global large scale switched system can be split into a set of smaller interconnected switched Takagi-Sugeno fuzzy subsystems. In this context, to stabilize the overall closedloop system, a set of switched non-Parallel-Distributed-Compensation (non-PDC) outputfeedback controllers is considered. The latter is designed based on Linear Matrix Inequalities (LMI) conditions obtained from a multiple switched non-quadratic Lyapunov-like candidate function. The controllers proposed herein are synthesized to satisfy $H \infty$ performances for disturbance attenuation. Finally, a numerical example is proposed to illustrate the effectiveness of the suggested decentralized switched controller design approach.


## Keywords

Large Scale Switched Fuzzy System, Decentralized non-PDC Controllers, Arbitrary Switching Laws.

## 1. INTRODUCTION

During the last few decades, several complex systems appeared to meet the specific needs of the world population. In this context, we can quote as examples networked power systems, water transportation networks, traffic systems, as well as other systems in various fields. Generally speaking, establish a mathematical model for large scale systems is a complex task, especially when the system is considered as a whole. Hence, to overcome these difficulties, an alternative to the global modelling approach has been explored. It consists in decomposing the overall large-scale system in a finite set of interconnected low-order subsystems [1].

Among these complex systems, switched interconnected large-scale system have attracted considerable attention since they provide a convenient modelling approach for many physical systems that can exhibit both continuous and discrete dynamic behavior. In this context, several studies dealing with the stability analysis and stabilization issues for both linear and nonlinear switched interconnected large-scale systems have been explored [1]-8]. Hence, the main challenge to deal with such problems consists in determining the conditions ensuring the stability of the whole systems with considera-
tion to the interconnections effects between its subsystems. Nevertheless, few works based on the approximation property of Takagi-Sugeno (TS) fuzzy models for nonlinear problems have been achieved to deal with the stabilization of continuous-time large-scale switched nonlinear systems [3], [8]-12].

The main interest of T-S models is their ability to accurately represent a nonlinear system as well as allowing to extend some of linear control concepts to nonlinear systems. To stabilize T-S models, the Parallel Distributed Compensation (PDC) control scheme is often considered. The basic philosophy of such control scheme is to design a controller sharing the same fuzzy membership functions structure as the T-S model to be controlled. Moreover, to reduce the conservatism of the design conditions, an extension of PDC contrrolers, called non-PDC controllers, can be considered with non-quadratic Lyapunov functions, or extended quadratic ones (see e.g. [13] 14 and references therein for more details).

In the context of T-S fuzzy switched largescale systems, an output-feedback decentralized PDC controller has been developed in 9]. In the same way, the authors of [10] have studied the design of an adaptive fuzzy output-feedback control for a class of switched uncertain nonlinear large-scale systems with unknown dead zones and immeasurable states. Recently, an observerbased decentralized control scheme was developed in 11 for a class switched non-linear largescale systems. In the same context, an adaptive fuzzy decentralized output-feedback tracking control has been explored in [12] for a class of switched nonlinear large-scale systems under the assumption that the large-scale system was composed of subsystems interconnected by their outputs. In this study, the stability of the whole closed-loop system and the tracking performance were achieved by using the Lyapunov function and under constrained switching signals with dwell time. However, such approaches may be restrictive since they are unsuitable in a more general case, i.e. when the switching sequences are arbitrary or unknown. Moreover, note that adaptive control approaches are based one parameter estimations. Therefore they often require more online computational capabilities
than robust control approaches, which can be a limitation for several embedded applications.

This paper presents the design of decentralized robust controllers for a class of switched TS interconnected large-scale systems with external bounded disturbances. More specifically, the primary contribution of this paper consists in proposing a LMI based methodology, in the non-quadratic framework, for the design of robust output-feedback decentralized switched non-PDC controllers for a class of large scale switched nonlinear systems under arbitrary switching laws. Moreover, to deals with external disturbances applied on the interconnected nonlinear subsystems, an criterion is considered. It aims at designing a robust controller, which attenuates the effects of the disturbances, which can be view as exogenous uncontrolled inputs, on the overall closed-loop dynamics.

The remainder of the paper is organized as follows. Section 2 presents the considered class of switched TS interconnected large-scale system, followed by the problem statement. The design of the decentralized switched non-PDC controllers is presented in section 3. A numerical example is proposed to illustrate the efficiency of the proposed approach in section 4. The paper ends with conclusions and references.

## 2. PROBLEM STATEMENT AND PRELIMIARIES

Let us consider the class of nonlinear hybrid systems $S$ composed of $n$ continuous time switched nonlinear subsystem $S_{i}$ represented by switched TS models. The $n$ state equations of the whole interconnected switched fuzzy system $S$
are given as follows; for $i=1,2, \ldots, n$ :

$$
\left\{\begin{array}{c}
\dot{x}_{i}(t)=\sum_{j_{i}=1}^{m_{i}} \sum_{j_{j_{i}}=1}^{r_{j_{i}}} \xi_{j_{i}}(t) h_{s_{j_{i}}}\left(z_{j_{i}}(t)\right)  \tag{1}\\
\cdot\left[\begin{array}{c}
A_{h j} x_{i}(t)+B_{h j} u_{i}(t)+B_{h j}^{w} w_{i}(t) \\
+\sum_{\alpha=1, \alpha \neq i}^{n}\left(F_{i, \alpha, h j} x_{\alpha}(t)+B_{h j}^{w_{\alpha}} w_{\alpha}(t)\right)
\end{array}\right] \\
y_{i}(t)=\sum_{j_{i}=1}^{m_{i}} \xi_{j_{i}}(t) C_{h j} x_{i}(t)
\end{array}\right.
$$

where $x_{i}(t) \in \mathbb{R}^{\eta_{i}}, y_{i}(t) \in \mathbb{R}^{\rho_{i}}, u_{i}(t) \in \mathbb{R}^{v_{i}}$ represent respectively the state, the measurement (output) and the input vectors associated to the $i^{\text {th }}$ subsystem. $w_{i}(t) \in \mathbb{R}^{v_{i}}$ is an uncontrollable time-varying $L_{2}$-norm bounded external disturbance associated to the $i^{\text {th }}$ subsystem. $m_{i}$ is the number of switching modes of the $i^{\text {th }}$ subsystem. $r_{j_{i}}$ is the number of fuzzy rules associated to the $i^{\text {th }}$ subsystem in the $j_{i}^{\text {th }}$ mode; for $i=1, \ldots, n, j_{i}=1, \ldots, m_{i}$ and $s_{j_{i}}=1, \ldots, r_{j_{i}}$, $A_{s_{j_{i}}} \in \mathbb{R}^{\eta_{i} \times \eta_{i}}, B_{s_{j_{i}}} \in \mathbb{R}^{\eta_{i} \times v_{i}}, B_{s_{j_{i}}}^{w} \in \mathbb{R}^{\eta_{i} \times v_{i}}$ and $C_{l_{j_{i}}} \in \mathbb{R}^{\rho_{i} \times \eta_{i}}$ are constant matrices describing the local dynamics of each polytopes; $B_{s_{j_{i}}}^{w_{\alpha}} \in \mathbb{R}^{\eta_{i} \times v_{\alpha}}$ and $F_{i, \alpha, s_{j_{i}}} \in \mathbb{R}^{\eta_{i} \times \eta_{\alpha}}$ express the interconnections between subsystems. $z_{j_{i}}(t)$ are the premises variables and $h_{s_{j_{i}}}\left(z_{j_{i}}(t)\right)$ are positive membership functions satisfying the convex sum proprieties $\sum_{s_{j_{i}}=1}^{r_{j_{i}}} h_{s_{j_{i}}}\left(z_{j_{i}}(t)\right)=1 ; \xi_{j_{i}}(t)$ is the switching rules of the $i^{\text {th }}$ subsystem, considered arbitrary but assumed to be real time available. These are defined such that the active system in the $l_{i}^{\text {th }}$ mode lead to:

$$
\left\{\begin{array}{l}
\xi_{j_{i}}(t)=1 \text { if } j_{i}=l_{i}  \tag{2}\\
\xi_{j_{i}}(t)=0 \text { if } j_{i} \neq l_{i}
\end{array}\right.
$$

Notations: In order to lighten the mathematical expression, one assumes the scalar $\underline{N}=\frac{1}{n-1}$, the index $i$ associated to the $i^{\text {th }}$ subsystem to denote the mode $j_{i}$. The premise entries $z_{j_{i}}$ will be omitted when there is no ambiguities and the following notation is employed for fuzzy matrices:

$$
G_{h j}=\sum_{s_{j_{i}}=1}^{r_{j_{i}}} h_{s_{j_{i}}} G_{s_{j_{i}}}
$$

and

$$
Y_{h_{j}, h_{j}}=\sum_{s_{j_{i}}=1}^{r_{j_{i}}} \sum_{k_{j_{i}}=1}^{r_{j_{i}}} h_{s_{j_{i}}} h_{k_{j_{i}}} Y_{s_{j_{i}}, k_{j_{i}}}
$$

Moreover, for matrices of appropriate dimensions we will denote: $\dot{X}_{h j}=\frac{d X_{h_{j_{i}}}}{d t}$ and $\left(\dot{X}_{h j}\right)^{-1}=\frac{d\left(X_{h_{j i}}\right)^{-1}}{d t}$. As usual, a star $\left({ }^{*}\right)$ indicates a transpose quantity in a symmetric matrix and $\operatorname{sym}(G)=G+G^{T}$. The time $t$ will be omitted when there is no ambiguity. However, one denotes $t_{j \rightarrow j^{+}}$the switching instants of the $i^{t h}$ subsystem between the current mode $j$ (at time $t$ ) and the upcoming mode $j^{+}$(at time $\left.t^{+}\right)$, therefore we have:

$$
\left\{\begin{array} { l } 
{ \xi _ { j } ( t ) = 1 }  \tag{3}\\
{ \xi _ { j ^ { + } } ( t ) = 0 }
\end{array} \text { and } \left\{\begin{array}{l}
\xi_{j}\left(t^{+}\right)=0 \\
\xi_{j^{+}}\left(t^{+}\right)=1
\end{array}\right.\right.
$$

In the sequel, we will deal with the robust output-feedback disturbance attenuation for the considered class of large-scale system $S$. For that purpose, a set of decentralized outputfeedback switched non-PDC control laws is proposed as; for $i=1, \ldots, n$ :

$$
\begin{equation*}
u_{i}(t)=\sum_{j_{i}=1}^{m_{i}} \xi_{j_{i}}(t) K_{h j}\left(X_{h j}^{9}\right)^{-1} y_{i}(t) . \tag{4}
\end{equation*}
$$

where the matrices

$$
\begin{aligned}
& K_{h j}=\sum_{k_{j_{i}}=1}^{r_{j_{i}}} h_{s_{j_{i}}}\left(z_{j_{i}}(t)\right) K_{k_{j_{i}}}, \\
& X_{h j}^{9}=\sum_{s_{j_{i}}=1}^{r_{j_{i}}} h_{s_{j_{i}}}\left(z_{j_{i}}(t)\right) X_{s_{j_{i}}}^{9}
\end{aligned}
$$

are the fuzzy gains to be synthesized with $X_{s_{j_{i}}}^{9}=$ $\left(X_{s_{j_{i}}}^{9}\right)^{T}>0$.
Remark 1: When a large scale system is considered as a whole, i.e. a high-order system, the size of the decision matrices (control gains, Lyapunov matrices...) in the LMI conditions increases the computational cost to check whether a solution exists. In this case, the available convex optimisation tools may fail to find a solution to the LMI problem (unfeasibility or computational crashes). This is mainly why the
decomposition of large-scale systems into lowerorder interconnected subsystems can be considered as a good alternative. Indeed, in this case, decentralized controllers design can be applied to each lower-order subsystem, i.e. with lowersized decision variables and LMIs, helping to reduce the computational workload of the convex optimization algorithms.

Substituting (4) into (1), one expresses the overall closed-loop dynamics $S_{c l}$ as, for $i=$ $1, \ldots, n$ :
$\dot{x}_{i}=\sum_{j=1}^{m_{i}} \xi_{j}\left\{\begin{array}{c}{\left[A_{h j}+B_{h j} K_{h j}\left(X_{h j}^{9}\right)^{-1} C_{h j}\right] x_{i}} \\ +\sum_{\alpha=1, \alpha \neq i}^{n} F_{i, \alpha, h j} x_{\alpha}\end{array}\right\}$

Thus, the problem considered in this study can be resumed as follows:
Problem 1: The objective is to design the controllers (4) such that the closed-loop interconnected large-scale switched TS system (5) satisfies a robust $\mathrm{H} \infty$ performance.

Definition 1: The switched interconnected large-scale system (1) is said to have a robust $\mathrm{H} \infty$ output-feedback performance if the following conditions are satisfied:

Condition 1 (Stability condition): With zero disturbances input condition $w_{i} \equiv 0$, for $i=1, \ldots, n$, the closed-loop dynamics (5) is stable.

Condition 2 (Robustness condition): For all non-zero $w_{i} \in L_{2}[0, \infty)$, under zero initial condition $x_{i}\left(t_{0}\right) \equiv 0$, it holds that for $i=1, \ldots, n$,

$$
\begin{align*}
& J_{i}=\int_{0}^{+\infty} x_{i}^{T} x_{i} d t \\
& \leqslant \varsigma_{i}^{2} \int_{0}^{+\infty}\left(w_{i}^{T} w_{i}+\sum_{\alpha=1, \alpha \neq i}^{n} w_{\alpha}^{T} w_{\alpha}\right) d t \tag{6}
\end{align*}
$$

where $\varsigma_{i}^{2}$ is a positive scalars which represents the disturbance attenuation level associated to the $i^{\text {th }}$ subsystem.
From the closed-loop dynamics (5), it can be seen that several crossing terms among the
gain controllers $K_{h j}$ and the system's matrices $\left(B_{h j} K_{h j}\left(X_{h j}\right)^{-1} C_{h j}\right)$ are present. Hence, in view of the wealth of interconnections characterizing our system, these crossing terms lead surely to very conservative conditions for the design of the proposed controller. In order to decouple the crossing terms $\left(B_{h j} K_{h j}\left(X_{h j}\right)^{-1} C_{h j}\right)$ appearing in the equation (5), and to provide LMI conditions, we use an interesting property called the descriptor redundancy [13]-16. In this context, the closed-loop dynamics (5) can be alternatively expressed as follows. First, from the output equation of (11) and the control law (4), we introduce null terms such that, for $i=1, \ldots, n$ :

$$
\begin{equation*}
0 \dot{y}_{i}=-y_{i}+C_{h j} x_{i} \tag{7}
\end{equation*}
$$

and:

$$
\begin{equation*}
0=u_{i}-K_{h j}\left(X_{h j}^{9}\right)^{-1} y_{i} \tag{8}
\end{equation*}
$$

Then, by considering the augmented state vectors $\tilde{x}_{i}^{T}=\left[\begin{array}{lll}x_{i}^{T} & y_{i}^{T} & u_{i}^{T}\end{array}\right], \tilde{x}_{\alpha}^{T}=\left[\begin{array}{lll}x_{\alpha}^{T} & y_{\alpha}^{T} & u_{\alpha}^{T}\end{array}\right]$ and disturbances $\tilde{w}_{i, \alpha}^{T}=\left[w_{i}^{T} w_{\alpha}^{T}\right]$, the closed-loop dynamics of the large-scale system (11) under the non-PDC controller (4) can be reformulated as follows, for $i=1, \ldots, n$ :

$$
\begin{align*}
E \dot{\tilde{x}}_{i} & =\tilde{A}_{h j, h j} \tilde{x}_{i} \\
& +\sum_{\alpha=1, \alpha \neq i}^{n}\left(\tilde{F}_{i, \alpha, h j} \tilde{x}_{\alpha}+\tilde{B}_{h j}^{w \alpha} \tilde{w}_{i, \alpha}\right) \tag{9}
\end{align*}
$$

$$
\begin{aligned}
& E=\left[\begin{array}{lll}
I & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right], \tilde{F}_{i, \alpha, h j}=\left[\begin{array}{ccc}
F_{i, \alpha, h j} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right] \\
& \tilde{B}_{h j}^{w \alpha}=\left[\begin{array}{cc}
\underline{N} B_{h j}^{w} & 0 \\
0 & B_{h j}^{w \alpha} \\
0 & 0
\end{array}\right] \\
& \tilde{A}_{h j, h j}=\left[\begin{array}{ccc}
A_{h j} & 0 & B_{h j} \\
0 & -I & K_{h j}\left(X_{h j}^{\gamma}\right)^{-1} \\
C_{h j} & 0 & -I
\end{array}\right]
\end{aligned}
$$

Note that the system (9) is a large scale switched descriptor. Hence, it is worth pointing out that the output-feedback stabilization problem of the system (1) can be converted into the stabilization problem of the augmented system (9).

Remark 2: It may be hard to work with the first formulation of the closed-loop dynamics (5), due to the large number of crossing terms. However, the goal of our study can now be achieved by considering the augmented closed-loop dynamics (9) expressed in the descriptor form. In this context, the second condition of Definition 1 , given by equation (6), can be reformulated as follows:

$$
\begin{equation*}
\int_{0}^{+\infty} \tilde{y}_{i}^{T} Q \tilde{y}_{i} d t \leqslant \varsigma_{i}^{2} \int_{0}^{+\infty} \sum_{\alpha=1, \alpha \neq i}^{n} \tilde{w}_{i, \alpha}^{T} \Xi \tilde{w}_{i, \alpha} d t \tag{10}
\end{equation*}
$$

with $\Xi=\left[\begin{array}{cc}\underline{N} I & 0 \\ 0 & I\end{array}\right], Q=\left[\begin{array}{lll}0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0\end{array}\right]$.
To conclude the preliminaries, let us introduce the following lemma, which will be used in the sequel.

Lemma 1. 17: Let us consider two matrices $A$ and $B$ with appropriate dimensions and a positive scalar $\tau$, the following inequality is always satisfied:

$$
\begin{equation*}
A^{T} B+B^{T} A \leqslant \tau A^{T} A+\tau^{-1} B^{T} B \tag{11}
\end{equation*}
$$

## 3. LMI Based Decentralized Controller Design

In this section, the main result for the design of a robust decentralized switched non-PDC controller (4) ensuring the closed-loop stability of (5) and the $\mathrm{H} \infty$ disturbance rejection performance (10) is presented. It is summarized by the following theorem.

Theorem 1. Assume that for each subsystem $i$ of (1), the active mode is denoted by $j_{i}$ and, for $j_{i}=1, \ldots, m_{i}$ and $s_{j_{i}}=1, \ldots, r_{j_{i}}$, $\dot{h}_{s_{j_{i}}}(z(t)) \geqslant \lambda_{s_{j_{i}}}$. The overall interconnected switched TS system (1) is stabilized by a set of $n$ decentralized switched non-PDC control laws (4) according to the Definition 1, if there exists, for all combinations of $i=1, \ldots, n, j_{i}=1, \ldots, m_{i}$, $j_{i}{ }^{+}=1, \ldots, m_{i}, s_{j_{i}}=1, \ldots, r_{j_{i}}, k_{j_{i}}=1, \ldots, r_{j_{i}}$, $k_{j_{i}}^{1}=1, \ldots, r_{j_{i}}$ and $l_{j_{i}}=1, \ldots, r_{j_{i}}$, the matrices $X_{k_{j_{i}}}^{1}=\left(X_{k_{j_{i}}}^{1}\right)^{T}>0, X_{k_{j_{i}}}^{5}=\left(X_{k_{j_{i}}}^{5}\right)^{T}>0 ;$
$X_{k_{j_{i}}}^{9}=\left(X_{k_{j_{i}}}^{9}\right)^{T}>0, W_{s_{j_{i}} s_{j_{i}} k_{j_{i}}}^{1}, K_{k_{j_{i}}}$, and the scalars, $\tau_{1, i}, \ldots, \tau_{i-1, i}, \tau_{i+1, i}, \ldots, \tau_{n, i}$ (excepted $\tau_{i, i}$ which don't exist since there is no interaction between a subsystem and himself), such that the LMIs described by (12), (13), (14) and (15) are satisfied.

$$
\begin{equation*}
X_{k_{j_{i}}^{\prime}}^{1}+W_{s_{j_{i}}} k_{j_{i}} l_{j_{i}}>0 \tag{12}
\end{equation*}
$$

$$
\begin{gather*}
{\left[\begin{array}{cc}
-\mu_{j_{i} \rightarrow j_{i}+}+X_{k_{j_{i}}}^{1} & X_{k_{j_{i}}}^{1} \\
X_{k_{j_{i}}}^{1} & -X_{k_{j_{i}}+}^{1}
\end{array}\right] \leqslant 0}  \tag{13}\\
{\left[\begin{array}{c|c}
\Gamma_{s_{j_{i}} l_{j_{i}} k_{j_{i}}} & (*) \\
\hline \overline{X_{k_{j_{i}}}} & \overline{\bar{I}}
\end{array}\right]<0} \tag{14}
\end{gather*}
$$

$\left(\right.$| $\Lambda_{s_{j_{2}} k_{j_{i}}}$ |  | $*$ |  |
| :---: | :---: | :---: | :---: |
| $X_{k_{j_{i}}}$ | $-\tau_{\alpha, i} I$ |  |  |
| $\left(\tilde{B}_{s_{j_{i}}, \alpha}^{w, \alpha}\right.$ | 0 | $\underline{N} s_{i}^{2} I$ | 0 |
| 0 |  | 0 | $-I$ |$)<0$

with

$$
\begin{aligned}
& \Lambda_{s_{j_{i}} l_{j_{i}} k_{j_{i}}}=\left[\begin{array}{ccc|c} 
& & \\
& \Gamma_{s_{j_{i}}} l_{j_{i}} k_{j_{i}} & & * \\
* \\
\hline X_{k_{j_{i}}}^{2} & 0 & 0 & -I
\end{array}\right] \\
& \tilde{B}_{k j}^{w \alpha}=\left[\begin{array}{cc}
\underline{N} B_{k j}^{w} & 0 \\
0 & B_{k j}^{w \alpha} \\
0 & 0
\end{array}\right] \\
& \Phi_{s_{j_{i}} l_{j_{i}} k_{j_{i}} k_{j_{i}}^{\prime}}=\sum_{l_{j_{i}}=1}^{r_{j_{i}}} \lambda_{l_{j_{i}}}\left(X_{l_{j_{i}}}^{1}+W_{s_{j_{i}} k_{j_{i}} k_{j_{i}}^{\prime}}\right), \\
& X_{k_{j_{i}}}=\left[\begin{array}{ccc}
X_{k_{j_{i}}}^{1} & 0 & 0 \\
0 & X_{k_{j_{i}}}^{5} & 0 \\
0 & 0 & X_{k_{j_{i}}}^{9}
\end{array}\right] \\
& \overline{\bar{I}}=(-1) \\
& \cdot \operatorname{diag}\left[\begin{array}{llllll}
\tau_{1, i} I & \ldots & \tau_{i-1, i} I & \tau_{i+1, i} I & \ldots & \tau_{n, i} I
\end{array}\right] \\
& \overline{\bar{X}}_{k_{j_{i}}}=\left[\begin{array}{llllll}
X_{k_{j_{i}}} & \cdots & X_{k_{j_{i}}} & X_{k_{j_{i}}} & \cdots & X_{k_{j_{i}}}
\end{array}\right]
\end{aligned}
$$

$$
\Gamma_{s_{j_{i}} l_{j_{i}} k_{j_{i}}}=\left[\begin{array}{c|c|c}
\operatorname{sym}\left(X_{k_{j_{i}}}^{1}\right. & A_{s_{j_{2}}}^{T} & \\
+\tau_{i, \alpha} F_{i, \alpha, s_{j_{i}}} F_{i, \alpha, s_{j_{i}}}^{T} & (*) & (*) \\
-\Phi_{s_{j_{i}} l_{j_{i}} k_{j_{i}} k_{j_{i}}^{\prime}} & \Gamma_{s_{j_{i}} l_{j_{i}} k_{j_{j}}} & (*) \\
\hline 0 & -\operatorname{sym}\left(X_{k_{j_{i}}}^{5}\right) & (*) \\
\hline X_{k_{j_{i}}}^{9}\left(B_{s_{j_{i}}}\right)^{T} & \\
+C_{s_{j_{i}}} X_{k_{j_{i}}}^{1} & \left(K_{l_{j_{i}}}\right)^{T} & -\operatorname{sym}\left(X_{k_{j_{i}}}^{9}\right)
\end{array}\right] .
$$

Proof. The present proof is divided in two parts corresponding to the Conditions 1 and 2 given in Definition 1.
Part 1 (Stability Condition 1, Definition 1): With zero disturbances input condition $\tilde{w}_{i, \alpha} \equiv$ 0 , for $i=1, \ldots, n$. Let us define the following multiple switched non-quadratic Lyapunov-like candidate functional:

$$
\begin{equation*}
V\left(x_{1}, x_{2}, \ldots, x_{n}\right)=\sum_{i=1}^{n} \sum_{j_{i}=1}^{m_{i}} \xi_{j_{i}} v_{j_{i}}\left(x_{i}\right)>0 \tag{16}
\end{equation*}
$$

where

$$
\begin{aligned}
v_{j_{i}} & =\tilde{x}_{i}^{T} E\left(X_{h j}\right)^{-1} \tilde{x}_{i} \\
& =\tilde{x}_{i}^{T} E\left(\sum_{s_{j_{i}}=1}^{r_{j_{i}}} h_{s_{j_{i}}} X_{s_{j_{i}}}\right)^{-1} \tilde{x}_{i}
\end{aligned}
$$

and with $E X_{h j}=X_{h j} E>0, X_{h j}=$ $\operatorname{diag}\left[\begin{array}{lll}X_{h j}^{1} & X_{h j}^{5} & X_{h j}^{9}\end{array}\right], X_{h j}^{1}=X_{h j}^{1 T}$.

The augmented system (9), and implicitly the closed-loop interconnected switched system (5), is asymptotically stable if:

$$
\begin{equation*}
\forall t \neq t_{j \rightarrow j^{+}}, \quad \dot{V}\left(x_{1}, x_{2}, \ldots, x_{n}\right)<0 \tag{17}
\end{equation*}
$$

and:

$$
\begin{equation*}
v_{j_{i}+}\left(t_{j \rightarrow j^{+}}\right) \leqslant \mu_{j \rightarrow j^{+}} v_{j_{i}}\left(t_{j \rightarrow j^{+}}\right) \tag{18}
\end{equation*}
$$

where $\mu_{j \rightarrow j^{+}}$are positive scalars.
First, let us focus on the inequalities 18 . Their aim is to ensure the global decreasing behavior of the Lyapunov-like function (16) at the switching time $t_{j \rightarrow j^{+}}$. These inequalities are verified if, for $i=1, \ldots, n, j_{i}=1, \ldots, m_{i}$, $j_{i}^{+}=1, \ldots, m_{i}$ and $s_{j_{i}}=1, \ldots, r_{j_{i}}$ :

$$
\begin{equation*}
E\left(X_{h j^{+}}\right)^{-1}-\mu_{j \rightarrow j^{+}} E\left(X_{h j}\right)^{-1} \leqslant 0 \tag{19}
\end{equation*}
$$

That is to say:

$$
\begin{equation*}
\left(X_{h j+}^{1}\right)^{-1}-\mu_{j \rightarrow j^{+}}\left(X_{h j}^{1}\right)^{-1} \leqslant 0 \tag{20}
\end{equation*}
$$

Left and right multiplying by $X_{h j}^{1}$, then using Schur complement, 20 is equivalent to:

$$
\left[\begin{array}{cc}
-\mu_{j \rightarrow j+} X_{h j}^{1} & X_{h j}^{1}  \tag{21}\\
X_{h j}^{1} & -X_{h j^{+}}^{1}
\end{array}\right] \leqslant 0
$$

Now, let us deal with (17), with the above defined notations, it can be rewritten as, $\forall t \neq$ $t_{j \rightarrow j^{+}}$:
$\sum_{i=1}^{n}\left[\operatorname{sym}\left(\dot{\tilde{x}}_{i}^{T} E\left(X_{h j}\right)^{-1} \tilde{x}_{i}\right)+\tilde{x}_{i}^{T} E\left(\dot{X}_{h j}\right)^{-1} \tilde{x}_{i}\right]$
$<0$.
Substituting (9) into (22), we can write, $\forall t \neq$ $t_{j \rightarrow j^{+}}$:

$$
\begin{equation*}
<0 \tag{23}
\end{equation*}
$$

From (11), the inequality (23) can be bounded by, $\forall t \neq t_{j \rightarrow j^{+}}$:

$$
\sum_{i=1}^{n}\left[\tilde{x}_{i}^{T}\left[\begin{array}{c}
\operatorname{sym}\left(\left(X_{h j}\right)^{-1} \tilde{A}_{h j, h j}\right)+E\left(\dot{X}_{h j}\right)^{-1} \\
+\sum_{\alpha=1, \alpha \neq i}^{n} \tau_{i, \alpha}\left(X_{h j}\right)^{-1} \tilde{F}_{i, \alpha, h j} \tilde{F}_{i, \alpha, h j}^{T}\left(X_{h j}\right)^{-1}
\end{array}\right] \tilde{x}_{i}\right]
$$

$$
\begin{equation*}
<0 \tag{24}
\end{equation*}
$$

Moreover, since
$\sum_{i=1}^{n} \sum_{\alpha=1, \alpha \neq i}^{n} \tau_{i, \alpha}^{-1} x_{\alpha}^{T} x_{\alpha}=\sum_{i=1}^{n} \sum_{\alpha=1, \alpha \neq i}^{n} \tau_{\alpha, i}^{-1} x_{i}^{T} x_{i}, \forall x_{i}$,
(24) is satisfied if, for $i=1, \ldots, n$ and $\forall t \neq t_{j \rightarrow j^{+}}$:

$$
\begin{align*}
& \operatorname{sym}\left(\left(X_{h j}\right)^{-1} \tilde{A}_{h j, h j}\right)+E\left(\dot{X}_{h j}\right)^{-1} \\
& +\sum_{\alpha=1, \alpha \neq i}^{n}\left[\begin{array}{r}
\left.\tau_{i, \alpha}\left(X_{h j}\right)^{-1} \tilde{F}_{i, \alpha, h j} \tilde{F}_{T, \alpha, h j}^{T}\left(X_{h j}\right)^{-1}\right] \\
+\tau_{\alpha, i}^{-1} I
\end{array}\right] \\
& <0 . \tag{25}
\end{align*}
$$

Note that $E X_{h j}=X_{h j} E>0$, left and right multiplying the inequalities (25) respectively by $X_{h j}$, the inequality (25) can be rewritten as:

$$
\begin{align*}
& \operatorname{sym}\left(\tilde{A}_{h j, h j} X_{h j}\right)+E X_{h j}\left(\dot{X}_{h j}\right)^{-1} X_{h j} \\
& +\sum_{\alpha=1, \alpha \neq i}^{n}\left[\tau_{i, \alpha} \tilde{F}_{i, \alpha, h j} \tilde{F}_{i, \alpha, h j}^{T}+\tau_{\alpha, i}^{-1} X_{h j} X_{h j}\right] \\
& <0 . \tag{26}
\end{align*}
$$

Now, the aim is to obtain the inequality (14) from (26). This can be achieved with usual mathematical developments. First, note that

$$
\begin{aligned}
-E\left(\dot{X}_{h j}\right)^{-1} & =E\left(X_{h j}\right)^{-1} \dot{X}_{h j}\left(X_{h j}\right)^{-1} \\
& \leqslant-\Phi_{s_{j_{i}} l_{j_{i}} k_{j_{i}} k_{j_{i}}^{\prime}}
\end{aligned}
$$

(see [13] for more details on similar developments). Then, to deals with the term $X_{h j} X_{h j}$, one applies the Schur complement. This ends that part of the proof.

Part 2 (Robustness Condition 2, Definition 1): For all non-zero $\tilde{w}_{i, \alpha} \in L_{2}(0, \infty)$, under zero initial condition $\tilde{x}_{i}\left(t_{0}\right) \equiv 0$, it holds for $i=1, \ldots, N$ :

$$
\begin{equation*}
\sum_{i=1}^{n}\left[\dot{v}_{i}+\tilde{x}_{i}^{T} Q \tilde{x}_{i}^{T}-\varsigma_{i}^{2} \sum_{\alpha=1, \alpha \neq i}^{N} \tilde{w}_{i, \alpha} \Xi \tilde{w}_{i, \alpha}^{T}\right]<0 \tag{27}
\end{equation*}
$$

which is equivalent to:

$$
\sum_{i=1}^{n}\left[\begin{array}{c}
\operatorname{sym}\left(\dot{\tilde{x}}_{i}^{T} E\left(X_{h j}\right)^{-1} \tilde{x}_{i}\right)  \tag{28}\\
+\tilde{x}_{i}^{T}\left(E\left(\dot{X}_{h j}\right)^{-1}+Q\right) \tilde{x}_{i} \\
-\varsigma_{i}^{2} \sum_{\alpha=1 ; \alpha \neq i}^{N} \tilde{w}_{i, \alpha}^{T} \Xi \tilde{w}_{i, \alpha}
\end{array}\right]<0
$$

Substituting (9) into (28), we can write, $\forall t \neq$ $t_{j \rightarrow j^{+}}$:
$\sum_{i=1}^{n}\left[\begin{array}{c}\tilde{x}_{i}^{T}\left[\begin{array}{c}\operatorname{sym}\left(\tilde{A}_{h j, h j}^{T}\left(X_{h j}\right)^{-1}\right) \\ +Q+E\left(\dot{X}_{h j}\right)^{-1}\end{array}\right] \tilde{x}_{i} \\ +\sum_{\alpha=1, \alpha \neq i}^{n}\left[\begin{array}{c}\operatorname{sym}\left(\tilde{x}_{\alpha}^{T} F_{i, \alpha, h j}^{T}\left(X_{h j}\right)^{-1} \tilde{x}_{i}\right) \\ -\varsigma_{i}^{T} \tilde{w}_{i, \alpha}^{T} \Xi \tilde{w}_{i, \alpha} \\ +\operatorname{sym}\left(\tilde{w}_{i, \alpha}^{T}\left(\tilde{B}_{h j}^{w \alpha}\right)^{T}\left(X_{h j}\right)^{-1} \tilde{x}_{i}\right)\end{array}\right]\end{array}\right]$
$<0$
From (11), the inequality (23) can be bounded by, $\forall t \neq t_{j \rightarrow j^{+}}$:

$$
\left[\begin{array}{c}
\tilde{x}_{i}^{T} Y^{*} \tilde{x}_{i} \\
+\sum_{\substack{\alpha=1, \alpha \neq i}}^{n} \operatorname{sym}\left(\tilde{w}_{i, \alpha}^{T}\left(\tilde{B}_{h j}^{w \alpha}\right)^{T}\left(X_{h j}\right)^{-1} \tilde{x}_{i}\right) \\
+\sum_{\substack{\alpha=1 \\
\alpha \neq i}}^{n}\left(\tau_{i, \alpha}^{-1} \tilde{x}_{\alpha}^{T} \tilde{x}_{\alpha}-\varsigma_{i}^{2} \tilde{w}_{i, \alpha}^{T} \Xi \tilde{w}_{i, \alpha}\right)
\end{array}\right]
$$

$$
\begin{equation*}
<0 \tag{30}
\end{equation*}
$$

where $Y^{*}=$

$$
\left[\begin{array}{c}
\operatorname{sym}\left(\left(X_{h j}\right)^{-1} \tilde{A}_{h j h j}\right)+Q+E\left(\dot{X}_{h j}\right)^{-1} \\
+\sum_{\alpha=1, \alpha \neq i}^{n} \tau_{i, \alpha}\left(X_{h j}\right)^{-1} \tilde{F}_{i, \alpha, h j} \tilde{F}_{i, \alpha, h j}^{T}\left(X_{h j}\right)^{-1}
\end{array}\right] .
$$

Since
$\sum_{i=1}^{n} \sum_{\alpha=1, \alpha \neq i}^{n} \tau_{i, \alpha}^{-1} x_{\alpha}^{T} x_{\alpha}=\sum_{i=1}^{n} \sum_{\alpha=1, \alpha \neq i}^{n} \tau_{\alpha, i}^{-1} x_{i}^{T} x_{i}, \forall x_{i}$ and $\forall t \neq t_{j \rightarrow j^{+}},(24)$ is satisfied if:

$$
\sum_{i=1}^{n} \sum_{\alpha=1, \alpha \neq i}^{n}\left(\begin{array}{c}
\tilde{x}_{i}^{T} Y^{* *} \tilde{x}_{i}  \tag{31}\\
\left.+\operatorname{sym}\left(\tilde{w}_{i, \alpha}^{T} \tilde{B}_{w, \alpha, h_{h_{i}}}\left(X_{h_{j_{i}}}\right)^{-1} \tilde{x}_{i}\right)\right) \\
-\varsigma_{i}^{2} \tilde{w}_{i, \alpha}^{T} \Xi \tilde{w}_{i, \alpha}
\end{array}\right)
$$

$<0$,
where $Y^{* *}=$
$\binom{\underline{N}\left(\operatorname{sym}\left(\left(X_{h j}\right)^{-1} \tilde{A}_{h j h j}\right)+Q+E\left(\dot{X}_{h j}\right)^{-1}\right)}{+\tau_{i, \alpha}\left(X_{h_{j_{i}}}\right)^{-1} \tilde{F}_{i, \alpha, h_{j_{i}}} \tilde{F}_{i, \alpha, h_{j_{i}}}^{T}\left(X_{h_{j_{i}}}\right)^{-1}+\tau_{\alpha, i}^{-1} I}$
The previous equation can be rewritten as follow:
$\left[\begin{array}{c}\tilde{x}_{i} \\ \tilde{w}_{i, \alpha}\end{array}\right]^{T}\left[\begin{array}{cc}\Upsilon_{h j, h j h j} & * \\ \left(\tilde{B}_{h j}^{w \alpha}\right)^{T}\left(X_{h j}\right)^{-1} & -\varsigma_{i}^{2} \Xi\end{array}\right]\left[\begin{array}{c}\tilde{x}_{i} \\ \tilde{w}_{i, \alpha}\end{array}\right]$
$<0$

With: $\Upsilon_{h j, h j, h j}=$

$$
\begin{aligned}
& \underline{N}\left(\operatorname{sym}\left(\left(X_{h j}\right)^{-1} \tilde{A}_{h j, h j}\right)+Q+E\left(\dot{X}_{h j}\right)^{-1}\right) \\
& +\tau_{i, \alpha}\left(X_{h j}\right)^{-1} \tilde{F}_{i, \alpha, h j} \tilde{F}_{i, \alpha, h j}^{T}\left(X_{h j}\right)^{-1}+\tau_{\alpha, i}^{-1} I .
\end{aligned}
$$

Left and right multiplying the inequalities (25) respectively by $\left[\begin{array}{cc}X_{h j} & 0 \\ 0 & I\end{array}\right]$ it yields for $i=1, \ldots, n$ and $\alpha=1, \ldots, n$ with $\alpha \neq i$ :

$$
\begin{align*}
& \left(\begin{array}{c|c}
\begin{array}{c}
\underline{N} s y m \\
\\
+\underline{N} X_{h j} Q X_{h j} \\
\\
+\underline{N} E X_{h j}\left(\dot{X}_{h j}\right)^{-1} X_{h j} \\
+\left(\tau_{i, \alpha} \tilde{F}_{i, \alpha, h j} \tilde{F}_{i, \alpha, h j}^{T}+\tau_{\alpha, i}^{-1} X_{h j} X_{h j}\right)
\end{array} & \\
\hline X_{h_{j_{i}}}\left(\tilde{B}_{h j}^{w \alpha}\right)^{T} & -\varsigma_{i}^{2} \Xi
\end{array}\right) \\
& <0 \tag{33}
\end{align*}
$$ property $-E\left(\dot{X}_{h j}\right)^{-1}=E\left(X_{h j}\right)^{-1} \dot{X}_{h j}\left(X_{h j}\right)^{-1}$, we can major the derivative $-E \dot{X}_{h j}$ by $-\Phi_{s_{j_{i}}} l_{j_{i}} k_{j_{i}} k_{j_{i}}^{\prime}$ and then apply the Schur complement.

Remark 3: In this paper, one suppose that the whole system $S$ is decomposed into $n$ interconnected subsystems $S_{i}, i=1,2, \ldots, n$. The deal is to ensure the robust control of each subsystem despite of the interconnection between him and the others subsystems. Hence, the global problem is divided to low-order problems. However, when subsystems are high-order, then LMIs are high dimensional matrix so it is hard to be solved using Matlab LMI toolbox.

## 4. Numerical example

This section is dedicated to illustrate the effectiveness of the proposed LMI conditions. We consider the following system composed of two interconnected switched TS subsystems given by:

## Subsystem 1:

$$
\left\{\begin{array}{c}
\dot{x}_{1}=\sum_{j_{1}=1}^{2} \sum_{s_{j_{1}}=1}^{2} \xi_{j_{1}} h_{s_{j_{1}}}\left[\begin{array}{c}
A_{s_{j_{1}}} x_{1}+B_{s_{j_{1}}} u_{1} \\
+B_{s_{j_{1}}}^{w} w_{1}+F_{1,2, s_{j_{1}}} x_{2} \\
+B_{s_{j_{1}}}^{w_{2}} w_{2}
\end{array}\right] \\
y_{1}=\sum_{j_{1}=1}^{2} \sum_{s_{j_{1}}=1}^{2} \xi_{j_{1}} h_{s_{j_{1}}} C_{s_{j_{1}}} x_{1} \tag{on}
\end{array}\right.
$$

In the mode 1 , the values of variables are given by: $A b_{1}=1, A a_{11}=-2.1, A a_{21}=-1.1$, $B b_{1}=-1.2, B a_{11}=0, B a_{21}=1.2, C a_{11}=-.1$, $C a_{12}=1, F b_{1}=0.01, F a_{11}=.01, F a_{21}=.1$, $w b_{1}=0.01, w a_{11}=-.01, w a_{21}=-.02, \alpha b_{1}=$ $0.01, \alpha a_{11}=.02, \alpha a_{12}=.01$.
In the mode 2, the values of variables are given by: $A b_{2}=0.2, A a_{12}=-2, A a_{22}=-3, B b_{2}=$ $-1.5, B a_{12}=1, B a_{22}=3, C a_{21}=1, C a_{22}=.1$, $F b_{2}=-0.01, F a_{21}=.2, F a_{22}=.02, w b_{2}=$ $-0.05, w a_{12}=-.05, w a_{22}=.01, \alpha b_{2}=-0.05$, $\alpha a_{21}=.04, \alpha a_{22}=.03$.
The membership functions: $h_{1_{1_{1}}}\left(x_{1}(t)\right)=$ $\sin ^{2}\left(x_{11}(t)\right), \quad h_{2_{1} 1}\left(x_{1}(t)\right)=\sin ^{2}\left(x_{12}(t)\right)$, $h_{i_{2_{1}}}\left(x_{1}(t)\right)=1-h_{i_{1_{1}}}\left(x_{1}(t)\right)$.

## Subsystem 2:

$$
\left\{\begin{array}{c}
\dot{x}_{2}=\sum_{j_{2}=1}^{2} \sum_{s_{j_{2}}=1}^{2} \xi_{j_{2}} h_{s_{j_{2}}}\left[\begin{array}{c}
A_{s_{j_{2}}} x_{2}+B_{s_{j_{2}}} u_{2} \\
+B_{s_{j_{2}}}^{w} w_{2}+F_{2,1, s_{j_{2}}} x_{1} \\
+B_{s_{j_{2}}}^{w_{1}} w_{1}
\end{array}\right] \\
y_{2}=\sum_{j_{2}=1}^{2} \sum_{s_{j_{2}}=1}^{2} \xi_{j_{2}} h_{s_{j_{2}}} C_{s_{j_{2}}} x_{2} \tag{35}
\end{array}\right.
$$

with $x_{2}=\left[\begin{array}{lll}x_{21} & x_{22} & x_{23}\end{array}\right]^{T}$,

$$
\begin{aligned}
A_{s j_{2}} & =\left[\begin{array}{ccc}
-2 & A b_{j} & 0 \\
0 & A a_{s j} & 0 \\
0 & .1 & -1.1
\end{array}\right], \\
B_{s j_{2}} & =\left[\begin{array}{ccc}
-.1 & .5 & .1 \\
-.01 & .5 & .01 \\
-.01 & B a_{s j} & .1
\end{array}\right], \\
C_{s j_{2}} & =\left[\begin{array}{ccc}
.01 & C a_{s j} & .1 \\
-1 & .1 & 1 \\
.1 & .1 & .1
\end{array}\right], \\
F_{s j_{2}} & =\left[\begin{array}{ccc}
.01 & .001 & F a_{s j} \\
.01 & .01 & F b_{j}
\end{array}\right], \\
B w_{1 s j_{2}} & =\left[\begin{array}{lll}
\alpha b_{j} & .05 & \alpha a_{s j} \\
.001 & .001 & .001 \\
.001 & .001 & .001
\end{array}\right], \\
B w_{s j_{2}} & =\left[\begin{array}{lll}
w b_{j} & .05 & w a_{s j} \\
.001 & .001 & .001 \\
.001 & .001 & .001
\end{array}\right],
\end{aligned}
$$

In the mode 1 , the values of variables are given by: $A b_{1}=2, A a_{11}=-1, A a_{21}=-1.1, B a_{11}=$ $.01, B a_{21}=.02, C a_{11}=-.1, C a_{12}=-.2$, $F b_{1}=0.1, F a_{11}=.2, F a_{21}=.02, w b_{1}=-0.01$, $w a_{11}=.01, w a_{21}=.001, \alpha b_{1}=-.01, \alpha a_{11}=$ $.01, \alpha a_{12}=.001$.


Fig. 1: Closed-loop state responses of the interconnected switched Takagi-Sugeno systems.

In the mode 2, the values of variables are given by: $A b_{2}=1, A a_{12}=-2, A a_{22}=-3, B a_{12}=$ $0.03, B a_{22}=0.04, C a_{21}=-.4, C a_{22}=-.3$, $F b_{2}=0.2, F a_{21}=F a_{22}=.4, w b_{2}=0.01$, $w a_{12}=.02, w a_{22}=.05, \alpha b_{2}=.01, \alpha a_{21}=$



Fig. 2: Outputs trajectories of the overall closed-loop interconnected switched Takagi-Sugeno system.
$.02, \alpha a_{22}=.05$ and the membership functions $h_{1_{1_{2}}}\left(x_{2}\right)=\sin ^{2}\left(x_{21}\right), h_{2_{1_{2}}}\left(x_{2}\right)=\sin ^{2}\left(x_{22}\right)$, $h_{i_{2}}\left(x_{2}\right)=1-h_{i_{12}}\left(x_{2}\right)$. Let us assume that each subsystem switches under within the frontier defined by $\mathrm{H}_{11}=0.9 x_{11}+x_{12}, \mathrm{H}_{12}=-0.2 x_{11}+$ $9 x_{12}, \mathrm{H}_{21}=-x_{21}+x_{22}$ and $\mathrm{H}_{22}=x_{21}-2 x_{22}$. The external disturbances $w_{1}$ and $w_{2}$ are considered as white noise sequences.

A set of decentralized switched controllers (4) is synthesized based on Theorem 1 via the Matlab LMI toolbox. To do so, the lower bounds of the derivatives of the membership functions are prefixed as $\lambda_{1_{1_{1}}}=\lambda_{1_{2_{1}}}=\lambda_{1_{1_{2}}}=\lambda_{1_{2_{2}}}=-6$, and the disturbance attenuation level by $\varsigma_{1}^{2}=1.7$, $\varsigma_{2}^{2}=1.5$. The solution of the proposed theorem leads to the synthesis of two decentralized non-PDC switched TS controllers (4) with the following gain matrices:
$1^{\text {rst }}$ TS switched sub-controller:

$$
\begin{aligned}
& K_{{11_{1}}}=K_{2_{1_{1}}}=10^{-2} *\left[\begin{array}{cc}
-9.04 & -0.72 \\
-0.72 & -4.21
\end{array}\right] \\
& K_{1_{2_{1}}}=K_{2_{2_{1}}}=10^{-2} *\left[\begin{array}{cc}
-15.37 & 5.90 \\
5.90 & -14.14
\end{array}\right] \\
& X_{1_{1_{1}}}^{9}=\left[\begin{array}{cc}
0.2427 & -0.1589 \\
-0.1589 & 0.1892
\end{array}\right], \\
& X_{2_{1_{1}}}^{9}=\left[\begin{array}{cc}
0.2494 & -0.1589 \\
-0.1589 & 0.1936
\end{array}\right],
\end{aligned}
$$

$$
\begin{aligned}
& X_{1_{2_{1}}}^{9}=\left[\begin{array}{cc}
0.2449 & -0.1056 \\
-0.1056 & 0.3855
\end{array}\right] \\
& X_{2_{2_{1}}}^{9}=\left[\begin{array}{ll}
0.2826 & -0.125 \\
-0.125 & 0.42
\end{array}\right]
\end{aligned}
$$

$2^{\text {sd }}$ TS switched sub-controller:

$$
\begin{aligned}
& K_{1_{1_{2}}}=K_{2_{1_{2}}}= \\
& {\left[\begin{array}{ccc}
-0.7586 & 0.3474 & 0.1388 \\
0.3475 & -0.6394 & 0.0852 \\
0.1389 & 0.0853 & -1.0686
\end{array}\right]} \\
& K_{1_{2_{2}}}=K_{2_{2_{2}}}= \\
& {\left[\begin{array}{ccc}
-0.8513 & 0.1899 & 0.0906 \\
0.1899 & -0.8347 & 0.0661 \\
0.0906 & 0.0661 & -0.9930
\end{array}\right]} \\
& X_{1_{1_{2}}}^{9}=\left[\begin{array}{ccc}
2.0615 & -1.4032 & -0.9036 \\
-1.4032 & 1.4636 & -0.0925 \\
-0.9036 & -0.0925 & 4.035
\end{array}\right] \\
& X_{2_{1_{2}}}^{9}=\left[\begin{array}{ccc}
2.0064 & -1.361 & -0.7552 \\
-1.361 & 1.5038 & -0.0975 \\
-0.7552 & -0.0975 & 3.7587
\end{array}\right] \\
& X_{1_{2_{2}}}^{9}=\left[\begin{array}{ccc}
2.1295 & -0.8043 & -0.2885 \\
-0.8043 & 2.1742 & -0.1705 \\
-0.2885 & -0.1705 & 2.9487
\end{array}\right] \\
& X_{1_{2_{2}}}^{9}=\left[\begin{array}{ccc}
2.1104 & -0.8093 & -0.2917 \\
-0.8093 & 2.1822 & -0.1628 \\
-0.2917 & -0.1628 & 2.9275
\end{array}\right]
\end{aligned}
$$

The close-loop subsystems' dynamics are shown in Fig. 1 and Fig. 2 for the initial states $x_{1}(0)=\left[\begin{array}{ll}2 & 2\end{array}\right]^{T}$ and $x_{2}(0)=$ $\left[\begin{array}{lll}-1 & 1.5 & -1\end{array}\right]^{T}$. Moreover, Fig. 3 and Fig. 4 shows the control signals as well as the switching modes' evolution. As expected, the synthesized decentralized switched controller stabilizes the overall large scale switched system composed of (33) and (34).

## 5. CONCLUSIONS

This study has focused on large scale switched nonlinear systems where each nonlinear mode has been represented by a fuzzy TS system. To ensure the stability of the whole system in closed-loop, a set of decentralized switched nonPDC controllers has been considered. Therefore, LMI based conditions for the design of decentralized controllers have been proposed through


Fig. 3: Control signal and switched laws' evolutions of the first subsystem.


Fig. 4: Control signal and switched laws' evolutions of the second subsystem.
the consideration of a multiple switched nonquadratic Lyapunov-like function candidate and by using the descriptor redundancy formulation. Finally, a numerical example has been proposed to show the effectiveness of the proposed approach. An extension of the proposed approach to general switched systems under asynchronous switches will be the focus of our future works.

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