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An Efficient Image Contrast Enhancement Method using Sigmoid Function and Differential Evolution

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Abstract. *Image enhancement is an adjusting process to make an image more appropriate for certain applications. The contrast enhancement is one of the most frequently used image enhancement methods. In this study, we introduce a new image contrast enhancement method using a link between sigmoid function and Differential Evolution (DE) algorithm. DE algorithm is performed to identify the parameters in sigmoid function so that they can maximize the measure of contrast. The experimental results show that the proposed method not only retains the original image features but also enhances the contrast effectively.*

Keywords

Sigmoid function, differential evolution, contrast, enhancement, image.

1. Introduction

The first attempt towards digital image recognition was the color-based algorithm (color histogram or color distributive features) [1]. Therefore, the color contrast enhancement is a very important step in image processing. It is applied in medical image processing, remote sensing, and other areas [2–4].

In literature, there are several techniques for image contrast enhancement. The simplest technique is the global stretching or normalization technique. Given an original image with the intensity values in the interval $[a, b]$, this method can normalize the intensity values to the new interval $[a', b']$ $(a' < a < b < b')$ that leads to an increase in image contrast. However, this method can only apply a linear scaling function to the image and make the enhancement is less harsh. The other technique which is commonly used is histogram equalization [5–7]. This method transforms a low contrast image to high contrast image by distributing the components of the histogram to cover a wide range of gray scale with approximately uniform distribution. Some related researches that can improve the performance of histogram equalization method such as bi-histogram equalization, multi-histogram equalization, contrast limited adaptive histogram equalization, histogram specification [8–12]. Nevertheless, the approaches using histogram equalization and its relevant technique has a drawback when the mean intensity value of the image is shifted to the middle gray-level of the intensity range and may be difficult for the human eye. Thus, histogram equalization based techniques are not useful in the cases where brightness preservation is required [13].

Two other techniques that are often performed in current are fuzzy rule-based contrast enhancement [14–16] and image contrast enhancement using sigmoid function [17–25]. The sigmoid function is a continuous nonlinear activation function. The name, sigmoid, is obtained from the fact that the function is "S" shaped. The S-function allows more flexible control for the given regions; moreover, Kannan et al. demonstrated the superiority of sigmoid function over other approaches including fuzzy rule-based contrast enhancement. Therefore, it can be claimed that using the sigmoid function is the state-of-art image contrast enhancement method.

However, one major drawback in image contrast enhancement method using sigmoid function is that its parameters as the constant c and threshold th have not been identified exactly. Although Kannan et al. [19], via their experiment on eight of sports images, recommended that $c = 10$ and should be performed, this result is not suitable when dealing with various types of images due to the lack of global contrast optimization.

In order to fill the researched gaps mentioned above, this paper proposes a new image contrast enhancement using sigmoid function and evolutionary technique. In particular, the choice of parameters c and th is converted by chromosome representation including 6 genes (c and th for each R, G, B scale, respectively) at first. There are many heuristic algorithms but the outstanding algorithm is differential evolution (DE). The DE algorithm [26], is next utilized to find the solution that can maximize the contrast measure. The DE algorithm is one of the most popular evolutionary techniques and outperforms both genetic algorithm (GA) and ant colony optimization (ACO) algorithm in the solution quality and convergence rate [27–31]. Furthermore, even though a few modified DE methods as IDE [32, 33], aeDE [34], ect., were proposed, DE is more stable in searching the global optimization problem. Moreover, DE has been proven to be efficient and robust for benchmark and real-world problems [35–38]. Therefore, in this paper, DE is used in searching the optimal parameters of the sigmoid function. Several examples performed for various image categories in this paper demonstrate that the proposed algorithm improves significantly the measure of contrast in comparison with previous studies.

The rest of this paper is organized as follows. A review of image contrast enhancement using sigmoid function and the differential evolution are presented in Section 2. The proposed method are presented in Section 3. Section 4 shows the numerical examples, and Section 5 is the conclusion.

2. Related work

2.1. Contrast enhancement using the sigmoid function

Sigmoid function [17] is a continuous nonlinear activation function. The name, sigmoid, is obtained from the fact that the function is "S" shaped that can be given as

$$f(x) = \frac{1}{1 + e^{-c.x}}, \; c > 0, \; x \in [-1, 1]. \quad (1)$$

To deal with the image contrast enhancement problem, we put $x = f(x, y)$ then we have the modified sigmoid function including the contrast and threshold value as follows.

$$\begin{aligned} g(x,y) &= \frac{1}{1 + e^{-c.(f(x,y)-th)}} \\ &= \frac{1}{1 + e^{c.(th-f(x,y))}} \end{aligned} \quad (2)$$

where $g(x, y)$ is the enhanced pixel value, c is the contrast factor, th is the threshold value and $f(x, y)$ is the original image pixel value. In summary, given a color image with RGB scale, the algorithm for image contrast enhancement using a modified sigmoid function is proposed as follows.

Algorithm 1.
Step 1. Input the image $f(x, y)$.
Step 2. Extract R, G, B planes of the image.

Step 3. Re-scale the color planes to the range of $[0, 1]$.
Step 4. For each plane, apply the equation to get the enhanced pixel values.
Step 5. Finally concatenate the enhanced R, G, B planes to get the enhanced output image.

In the above algorithm, by adjusting the contrast factor and threshold value, it is possible to tailor the amount of lightening and darkening to control the overall contrast enhancement. The threshold value th is between in 0 and 1 and reaches the optimal value between 0.3 and 0.5, according to Kannan. Similarly, c is identified by 10, it is not completely exact in fact. According to our experiment presented below, the value of c is between 9.8 and 10 and cannot be identified unless using the evolutionary algorithm.

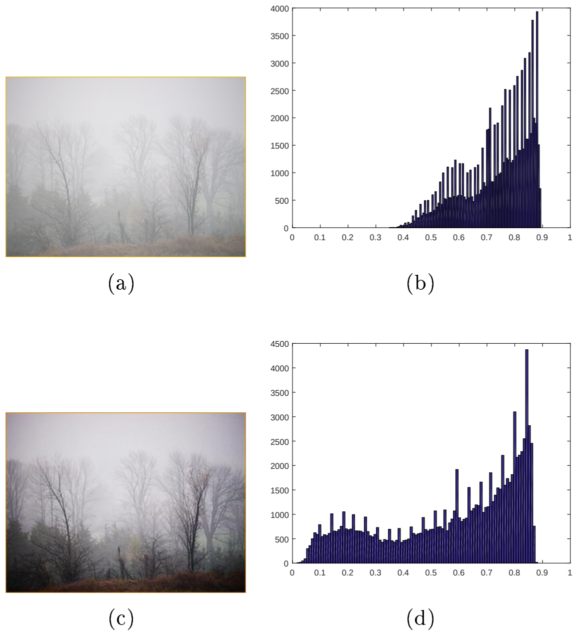


Fig. 1: Low, high contrast images and their histograms.

2.2. Image enhancement quality

1) Root Mean Square (RMS)

The contrast of an image is calculated by the luminance difference between its pixels. The high contrast image always has more luminance difference than low contrast image. This paper uses the RMS contrast [36] as the objective function for maximizing. Given the image of size $M \times N$, the RMS contrast is computed as follows.

$$RMS = \sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{L_{ij} - \bar{L}}{MN}} \tag{3}$$

where L_{ij} is the luminance of the pixel (i, j), $\overline{L}$ is the mean of luminance in the image. RMS contrast can be considered as the standard deviation of the pixel luminance in the image. For instance, in Fig. 1, it is clearly seen that the more contrast image, a larger standard deviation in histogram, and vice versa. Therefore, to enhance the image contrast, the RMS value needs to be maximized.

2) Effective Measure of Enhancement (EME)

The EME [39], a measure of image enhancement, is based on the Weber's and Fechner's laws. Let an image $f(x, y)$ be split into a number of blocks and using the equation

$$EME = \frac{1}{k_1 k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2} 20\log\left(\frac{I_{\max}^{i,j}}{I_{\min}^{i,j}}\right), \tag{4}$$

where k_1 and k_2 are the number of horizontal and vertical, respectively, blocks in the image $f(x, y)$; $I_{max}^{(i,j)}$, and $I_{min}^{(i,j)}$ are the maximum and minimum pixel values in a given block.

3) Absolute Measure of Enhancement (AME)

The AME [40] uses the relationship between the spread and the sum of the two luminance values found in a small block and the average value of the measured results of all blocks in the whole image. Let an image $f(x, y)$ be split into a number of blocks and using the equation

$$AME = -\frac{1}{k_1 k_2}\sum_{i=1}^{k_1}\sum_{j=2}^{k_2} 20\log\left(\frac{I_{\max}^{i,j} - I_{\min}^{i,j}}{I_{\max}^{i,j} + I_{\min}^{i,j}}\right), \tag{5}$$

where k_1 and k_2 are the number of horizontal and vertical, respectively, blocks in the image

$f(x,y)$; $I_{max}^{(i,j)}$ and $I_{min}^{(i,j)}$ are the maximum and minimum pixel values in a given block.

It can be worth noted that for RMS and EME, large values correspond to good image quality, whereas for AME, good quality corresponds to small values.

3. The proposed method

As mentioned before, the major drawback in image contrast enhancement method using the sigmoid function is that its parameters as the constant c and threshold th have not been identified exactly. Although Kannan et al. [19], via their experiment on eight of sports images, recommended that $c = 10$ and this result is not suitable when dealing with various types of images due to the lack of global contrast optimization. To fill the mentioned researched gap, DE algorithm is applied. That means the contrast image enhancement is now converted into the optimal problem. The objective function is to maximize the measure of contrast. The design variables are the value of c and th for each plane including Red (R), Green (G), and Blue (B). The representation for chromosome and objective function are briefly discussed below.

3.1. Chromosome Representation

As mentioned before, the modified sigmoid function is applied to enhance the pixel value of each plane; therefore there are six variables including c, th for R plane, c, th for G plane and c, th for B plane should be identified. To transform the problem of contrast enhancement into the optimization problem, each candidate solution is encoded into a vector of the $\mathbf{x} = \{x_1, x_2, \ldots, x_6\}$ chromosome at first. In the mentioned chromosome, x_1 and x_2 are the corresponding contrast factor c and threshold th of R plane. Similarly, x_3 and x_4 are the corresponding contrast factor c and threshold th of G plane; x_5 and x_6 are the corresponding contrast factor c and threshold th of B plane. For example, the recommended solution of Kannan with $c = 10$ and $th = 0.3$ for all planes is represented as

$$\begin{array}{cccccccc} \mathbf{x} & = & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ & = & 10 & 0.3 & 10 & 0.3 & 10 & 0.3 \end{array}$$

3.2. Differential evolution algorithm

After encoding each solution into a vector of the chromosome and establishing the objective function, the differential evolution algorithm [26] is adopted to maximize the objective function.The DE is a well-known global search method based on population, designed to deal with continuous optimization problems. There are four major steps in the procedure of DE including initialization, mutation, crossover, and selection.

Initialization Initially, a population with NP individuals is created by random sample from the feasible space. In case of image enhancement using sigmoid function, each individual is a vector consisting of six design variables $\mathbf{x}_i = \{x_{i,1}, x_{i,2}, x_{i,3}, \ldots, x_{i,6}\}$ defined as:

$$\begin{aligned} x_{i,j} &= x_j^l + \text{rand}[0,1] \times \left(x_j^u - x_j^l\right), \\ & i = 1,2,...,NP;\ j = 1,2,...,n \end{aligned} \quad (6)$$

where x_j^l and x_j^u are respectively the lower and upper bounds of ; rand $[0,1]$ is the real number having the uniform distribution within $[0,1]$; NP is the population size. Note that, as mentioned in Subsection 3.1. , the lower and upper bounds of $\mathbf{x}$ are defined by two following chromosomes.

$$\begin{array}{cccccccc} \mathbf{x^l} & = & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ & = & 1 & 0 & 1 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{cccccccc} \mathbf{x^u} & = & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ & = & 10 & 1 & 10 & 1 & 10 & 1 \end{array}$$

Mutation Next, a mutant vector $\mathbf{v}_i$ is generated by individuals' $\mathbf{x}_i$ in the population through mutation operations. Some mutation operations are regularly used in the DE as:

- rank/1: $\mathbf{v}_i = \mathbf{x}_{r_1} + F \times (\mathbf{x}_{r_2} - \mathbf{x}_{r_3})$,
- rank/2: $\mathbf{v}_i = \mathbf{x}_{r_1} + F \times (\mathbf{x}_{r_2} - \mathbf{x}_{r_3}) + F \times (\mathbf{x}_{r_4} - \mathbf{x}_{r_5})$,

- best/1:
  $\mathbf{v}_i = \mathbf{x}_{best} + F \times (\mathbf{x}_{r_1} - \mathbf{x}_{r_2})$,
- best/2:
  $\mathbf{v}_i = \mathbf{x}_{best} + F \times (\mathbf{x}_{r_1} - \mathbf{x}_{r_2}) + F \times (\mathbf{x}_{r_3} - \mathbf{x}_{r_4})$,
- current-to-best/1:
  $\mathbf{v}_i = \mathbf{x}_i + F \times (\mathbf{x}_{best} - \mathbf{x}_i) + F \times (\mathbf{x}_{r_1} - \mathbf{x}_{r_2})$,

where integers r_1, r_2, r_3, r_4, r_5 are randomly selected from $\{1, 2, \ldots, NP\}$ and must satisfy $r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq i$; F is the scale factor and randomly chosen within $[0, 2]$; $\mathbf{x}_{best}$ is the best individual in the current population.

After mutation, in the case of the j^{th} component v_{ij} of mutant vector $\mathbf{v_i}$ violates its boundary constraints, it will be reflected back to allowable region as described in following formula:

$$v_{ij} = \begin{cases} 2x_j^l - v_{ij} & \text{if} \quad v_{ij} < x_j^l, \\ 2x_j^u - v_{ij} & \text{if} \quad v_{ij} > x_j^u, \\ v_{ij} & \quad\quad \text{otherwise.} \end{cases} \tag{7}$$

Crossover After completing the mutation, each target vector $\mathbf{x_i}$ produces a trial vector $\mathbf{u_i}$ by substituting some components of the vector $\mathbf{x_i}$ by some components of the mutant vector $\mathbf{v_i}$ through the following binomial crossover operation.

$$u_{ij} = \begin{cases} v_{ij} & \text{if rand}[0,1] \leq CR \text{ or } j = j_{rand}, \\ x_{ij} & \text{otherwise.} \end{cases} \tag{8}$$

where $i \in \{1, 2, \ldots, NP\}$; $j \in \{1, 2, \ldots, 6\}$; j_{rand} is the integer selected in range $[1, 6]$; and CR is the crossover control parameter chosen within $[0, 1]$.

Selection Finally, each trial vector $\mathbf{u_i}$ is compared to its target vector $\mathbf{x_i}$. The better one with lower objective function value will serve as a new target vector $\mathbf{x_i}$ in the next generation.

$$\mathbf{x}_i = \begin{cases} \mathbf{u}_i & \text{if } f(\mathbf{u}_i) \leq f(\mathbf{x}_i), \\ \mathbf{x}_i & \text{otherwise.} \end{cases} \tag{9}$$

The DE stop searching when the absolute difference between the current optimum objective function and the mean of objective functions is less than a fixed value of tolerance. The whole process of image contrast enhancement using the sigmoid function and the DE is illustrated in Fig. 2. At the beginning, NP individuals are randomly initialized, with respect to upper and lower bound constraints. Through the process of Mutation and Crossover, we can create $2NP$ individuals consisting of NP old individuals (target vectors) and NP new individuals (trial vectors). Corresponding to these $2NP$ individuals, we get $2NP$ sets of parameters containing c and th values for each color channel R, G, B. Applying these parameter sets to the original image f we find $2NP$ new image g. In the selection process, the better NP sets, which result in better objective function values, will be selected through the next iterations. The above process is repeated until the stop condition is satisfied. Finally, the parameter set and the image with the best objective function are considered as the result of the algorithm.

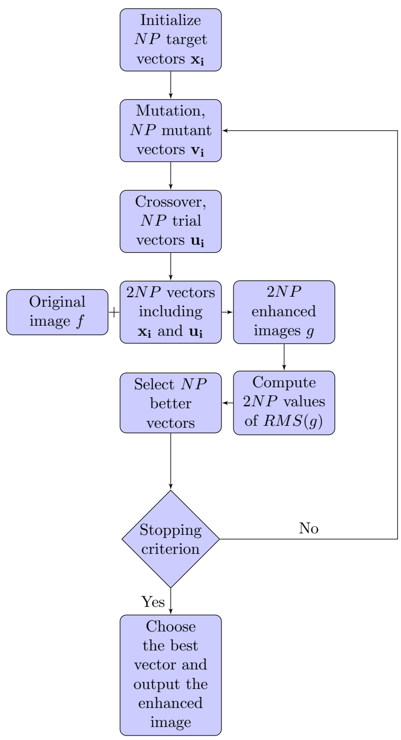


Fig. 2: Flowchart of the whole proposed algorithm.

4. Experiments

In this section, two numerical examples are carried out to present the proposed approach advantages. Example 1 step-by-step illustrates the proposed algorithm through the experiment on the well-known image "Lena". The experiment on the SCA-30 dataset [41,42] is presented in Example 2. In this example, we compare the performance of the new method and three alternative techniques consisting of the modified sigmoid function [19], the fuzzy-based approach with Gaussian membership function, and the Adaptive Histogram Equalization (ADE) [5]. The parameters of the DE algorithm are summarized in Tab. 1. For the Mutation Factor (F), according to [43], the higher value of F is, the greater of cost-effectiveness calculation of the global optimum's reliability is. For Crossover Probability (CR), the calculated performance of DE will be insensitive if CR belongs to $[0, 0.1]$ or $[0.9, 1]$ intervals. Therefore, we choose $F = 0.8$ and $CR = 0.9$, respectively. In addition, a $NP = 5 * dim$ has been recommended. In the solved problem, the number of dimensions is 6, hence, a population size of 30 is chosen. To improve the accuracy of the results, we reduced the tolerance to 0.1%, and the maximum number of iterations is 500 with DE/rand/1 mutation operator.

Tab. 1: Parameters of DE.

Parameter	**Value**
Mutation factor (F)	0.8
Crossover factor (CR)	0.9
Mutation operator	rand/1
Max iteration	500
Population size	30
Tolerance	1e-3

Example 1 In this example, the well-known image "Lena" is used as an experiment to illustrate the details of the proposed method. The original image is extracted to three planes R, G, B at first. The DE is next utilized to reach the optimal parameter solutions. The convergence of the algorithm is presented in Fig. 3. According to it, the optimal parameters are represented

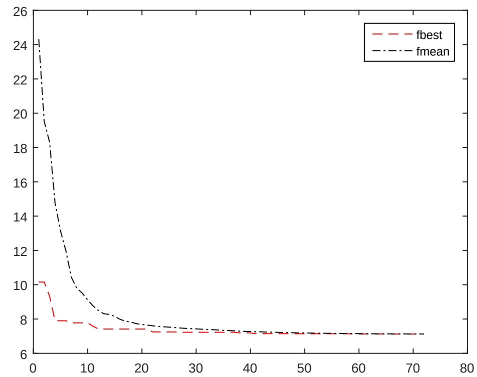


Fig. 3: The convergence of the proposed algorithm.

by the chromosome:

$$\mathbf{x} = 9.998\ 0.516\ 9.998\ 0.437\ 9.958\ 0.454$$

It means the RMS contrast of "Lena" can be maximized when using the sigmoid function with parameters $(c, th) = (0.998, 0.516), (0.998, 0.437), (0.958, 0.454)$ for R, G, B, respectively. It can be seen that the new method result is similar to the recommendation of Kannan but more flexible. Specifically, the parameters are not fixed at $c = 10$ and $th = 0.3$ but must be found correctly. Also, the parameter values in the three planes are not required to be the same. The enhanced results of comparative methods are presented in Tab. 2. From Tab. 2, it can be seen that the enhanced image created by the proposed method is more visible than images of other methods; also, the RMS contrast of proposed methods is the largest. It demonstrates the superiority of the proposed method over others in both qualitative and quantitative assessment.

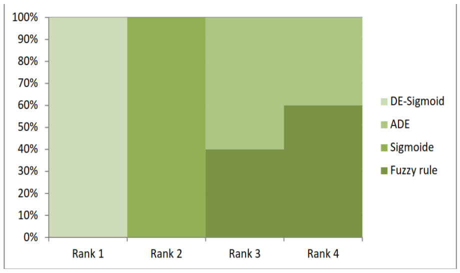
Example 2 This example tests the performance of the proposed method through the experiment on the SCA-30 image dataset. This dataset includes 30 real-world images captured with different cameras, under different lighting conditions. In this example, the comparison result between the Sigmoid function-based approach [19], the fuzzy-based approach with Gaussian membership function, the Adaptive Histogram Equalization (ADE) [5], and the proposed method (DE-Sigmoid) is displayed. Table

Tab. 2: The results of enhancement methods.

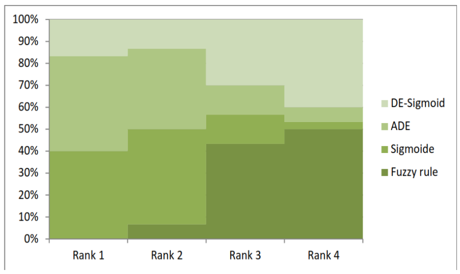
	Fuzzy rule	Sigmoid function	DE-sigmoid function
			
RMS contrast	0.0417	0.092	0.1404

3 shows some examples of enhancement. For the six illustrated images, comparative methods performance in terms of RMS contrast, EME, AME, and ranks is presented in Tab. 4. The performance on the whole SCA-30 image dataset is shown in Fig. 4.

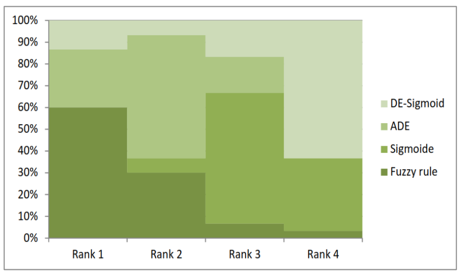
Table 4 and Fig. 4 demonstrate the superiority of the DE-Sigmoid over the comparative methods in terms of RMS. For the EME and AME measures, the DE-Sigmoid also presents a competitive performance. Furthermore, as shown in Tab. 3, the images provided by the proposed method are visible to human eye. The proposed method can clarify the small details that are almost hidden in the original images. For the computational cost, it takes some minutes for the optimization process using the DE algorithm. This is slower than the other methods. Based on the obtained results, the proposed method can be considered as a potential algorithm and can be useful for some specific problems in which the time is not a very important problem. Besides, adjusting the DE's parameters to trade off the improved performance for the computational cost is necessary to be further studied.



(a) Rank in terms of RMS.



(b) Rank in terms of EME.



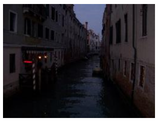
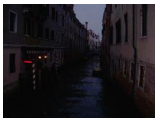
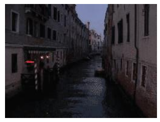
(c) Rank in terms of AME.

Fig. 4: Performance of comparative methods on SCA-30 dataset.

5. Conclusion

This paper proposes a method for contrast enhancement using sigmoid function and DE algorithm. In particular, DE is utilized to search the optimal threshold th and optimal contrast factor c in each color plane. The numerical examples show that the DE-Sigmoid outperforms the other comparative algorithms in terms of RMS contrast, and competes the others in terms of EME and AME. The proposed method has a disadvantage when the improved performance is roughly proportional to the computational cost. Therefore, it can be useful for some specific problems where users need to enhance the overall contrast of images, and the processing time is not a very important problem.

Tab. 3: The original and enhanced images of comparative methods.

	Original image	Fuzzy rule	Sigmoid	ADE	DE-sigmoid
Image 1					
Image 2					
Image 3					
Image 4					
Image 5					
Image 6					

Tab. 4: The performance on illustrated images.

		Fuzzy Rule	Sigmoid	ADE	DE-Sigmoid
Image 1	RMS (Rank)	0.239 (4)	0.349 (2)	0.241 (3)	0.350 (1)
	EME (Rank)	5.517 (4)	6.281 (2)	5.547 (3)	6.591 (1)
	AME (Rank)	15.597 (1)	19.351 (4)	16.609 (2)	17.647 (3)
Image 2	RMS (Rank)	0.214 (4)	0.310 (2)	0.217 (3)	0.325 (1)
	EME (Rank)	10.706 (3)	12.117 (2)	10.537 (4)	13.265 (1)
	AME (Rank)	27.438 (2)	31.94 (4)	28.244 (3)	25.998 (1)
Image 3	RMS (Rank)	0.120 (4)	0.206 (2)	0.153 (3)	0.238 (1)
	EME (Rank)	6.745 (4)	7.086 (3)	8.039 (2)	12.471 (1)
	AME (Rank)	48.043 (3)	58.002 (4)	45.933 (1)	46.234 (2)
Image 4	RMS (Rank)	0.101 (4)	0.163 (2)	0.118 (3)	0.198 (1)
	EME (Rank)	3.425 (3)	3.670 (2)	4.006 (1)	3.342 (4)
	AME (Rank)	7.366 (1)	10.976 (3)	10.064 (2)	19.199 (4)
Image 5	RMS (Rank)	0.078 (4)	0.118 (2)	0.092 (3)	0.152 (1)
	EME (Rank)	5.371 (3)	5.809 (2)	6.874 (1)	5.079 (4)
	AME (Rank)	11.641 (1)	18.281 (3)	15.821 (2)	35.055 (4)
Image 6	RMS (Rank)	0.142 (4)	0.22 (2)	0.154 (3)	0.235 (1)
	EME (Rank)	5.316 (4)	5.743 (2)	6.076 (1)	5.433 (3)
	AME (Rank)	6.658 (1)	9.679 (3)	7.858 (2)	13.234 (4)

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