Exploring the Effect of Peclet Number on the Transient Thermal Response of a Convective-Radiative Moving Porous Fin

M. G. SOBAMOWO[1,], A. A. YINUSA[1], H. BERREHAL[2], R. O. OLA-GBADAMOSI[3], M. O. SALAMI[1], E. H. ABUBAKAR[4], S. A. OLADOSU[3]*

[1]Department of Mechanical Engineering, University of Lagos, Akoka, Lagos, Nigeria
[2]Department of Physics, University of Brothers Mentouri Constantine 1, Algeria
[3]Department of Mechanical Engineering, Lagos State University, Epe Campus, Lagos, Nigeria
[4]Department of Civil and Environmental Engineering, University of Lagos, Akoka, Lagos, Nigeria

*Corresponding Author: M. G. SOBAMOWO (Email: mikegbeminiyiprof@yahoo.com)
(Received: 13-Oct-2022; accepted: 24-Feb-2023; published: 31-Mar-2023)
DOI: http://dx.doi.org/10.55579/jaec.202371.392

Abstract. *In the thermal analysis of moving fins, there have been conflicting results as well as discussion on the effects of Peclet number on the thermal responses of fins. While some authors agreed the increase in Peclet number, increases the fin temperature, the other class of researchers is of the opinion that when the Peclet number increases, the fin temperature decreases. It could be said that many of these divergence views arose from the physics of the problem as well as the mathematical model governing the heat transfer problem. Therefore, in this work, through modeling from the first principle, the effect of Peclet number on the thermal behaviour of convective-radiative moving porous fin is explored. First, a transient thermal model of a convective-radiative rectangular moving porous fin with temperature-dependent internal heat generation is developed. The developed thermal model is nondimensionalized to bring up the needed Peclet number in the adimensional governing equation of the heat transfer process. Thereafter, the model is solved analytically using the Laplace transform method and the effect of Peclet number on the thermal behaviour of the fin is investigated and discussed. It is hoped that the present study will help for a better understanding of the thermal problems in extended surfaces.*

Keywords

Moving fin, peclet number, porous fin, thermal analysis, transient response.

1. Introduction

In order to augment the rate of heat transfer on thermal and electronic components, fins are widely used as a passive method. In practice, the extended surfaces are attached to heat transfer devices and components to facilitate the rate of heat transfer from the prime surface. Such an important passive method of heat transfer enhancements has provoked several studies over the past decades [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. The study of thermal behavior of continuously moving surfaces such as extrusion, hot rolling, glass sheet or wire drawing, casting, powder metallurgy techniques for the fabrication of rod and sheet have become an area of increasing research interest. In the processes such as rolling of strips, hot rolling, glass fiber drawing, casting, extrusion, and drawing of sheets and wires, there is usually the presence of heat exchange between the surrounding and the stationary or moving material as depicted in Fig. 1 where hot plate/billet emerges from a die or furnace.

Since the schematic depicted in Figure 1 satisfies the approximate working condition of a heat-exchanging device, they can be modeled as fins moving uninterruptedly. Due to these adaptable and wide areas of applications, there have been extensive research works on the continuous moving fins. Moreover, in industrial processes, control of the cooling rate of the sheets is very important to obtain desired material structure. As a result, numerous works on thermal investigation of moving fins have been offered in previous studies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. Various heat transfer techniques in variable thermal conductivity moving fin with and without heat generation have been presented [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. However, scanning through all the reviewed published works as presented in literature, there are different and opposing results as well as discussion on the effect of Peclet number on the thermal responses of fins. Such conflicting results and discussions show a great deal of disagreement even among the titans of heat transfer enhancements by the passive device. The present study seeks to explore the mathematics and the physics of the heat transfer problem in order to establish some facts among the existing different schools of thought on the thermal problem. Therefore, in this work, a transient thermal model of a convective-radiative rectangular moving porous fin with temperature-dependent thermal conductivity and internal heat generation is developed and presented. The developed thermal model is nondimensionalized to bring up the needed Peclet number in the adimensional governing equation of the heat transfer process. Thereafter, the model is solved analytically using the Laplace transform method and the effect of Peclet number on the thermal behaviour of the fin is discussed.

2. Model Development for the Transient Thermal Flow Process

Fig. 1 presents an internally heated longitudinal porous moving fin of length L, thickness δ and perfectly and thermally attached to a prime surface at temperature Tb. Assuming that the porous fin tip is considered under adiabatic condition and the porous medium with fin material is homogeneous and isotropic for the unidirectional heat flow along the fin length. Also, local thermodynamic equilibrium prevails between the porous medium and the saturated with a single-phase fluid. The thermophysical properties of the fin material and the fluid are constant, and the fluid density variation follows Boussinesq approximation.

Using the assumptions stated above, the thermal energy balance equation of the extended surface is given by the following equation

$$\begin{aligned} &Energyinleftface + heatgeneratedinelement \\ &\quad = energyoutrightface \\ &\qquad + energylostbyconvection \\ &\qquad + energylostbyimmersedfluid \\ &\qquad + energylostbyradiation \\ &\qquad + energylostbymoving + accumulatedheat \end{aligned} \tag{1}$$

$$\begin{aligned} &q_x + q'''(T)\,A_{cr}dx = \left(q_x + \frac{\delta q}{\delta x}dx\right) + h(T)\,A_{suf}(T - T_a)(1-\phi) + \dot{m}c_p(T - T_a) \\ &+ \sigma\varepsilon(T)\,A_{suf}(T^4 - T_s^4) + (\rho c_p)_{eff}A_{cr}u\frac{\partial T}{\partial x}dx + (\rho c_p)_{eff}A_{cr}\frac{\partial T}{\partial t}dx \end{aligned} \tag{2}$$

Eq. (2) can be written as

$$q_x + (1-\phi)\,q'''(T)\,A_{cr}dx = \left(q_x + \frac{\delta q}{\delta x}dx\right) + hP(T-T_a)\,(1-\phi)\,dx + \dot{m}c_p\,(T-T_a) + \sigma\varepsilon P(T^4 - T_s^4)dx + (\rho c_p)_{eff}A_{cr}u\frac{\partial T}{\partial x} + (\rho c_p)_{eff}A_{cr}\frac{\partial T}{\partial t}dx \tag{3}$$

The rate of flow of fluid through the porous medium is given by

$$\dot{m} = \phi\rho_f V_w W dx \tag{4}$$

While the fluid flow velocity is and

$$V_w = \frac{gK\beta}{v}\,(T-T_a) \tag{5}$$

Therefore, after the substitution of Eq. (5) into Eq. (4), the mass flow rate of the fluid is given as

$$\dot{m} = \frac{\rho_f gK\beta W\phi}{v}\,(T-T_a)\,dx \tag{6}$$

The introduction of Eq. (7) into Eq. (3) produces Eq. (7)

$$\begin{aligned}
&-\frac{\partial q}{\partial x} + (1-\phi)\,q'''(T)\,A_{cr}dx \\
&= hP\,(1-\phi)\,(T-T_a)dx \\
&\quad + \frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{v}dx \\
&\quad + \sigma\varepsilon P(T^4 - T_s^4)dx + (\rho c_p)_{eff}A_{cr}u\frac{\partial T}{\partial x} \\
&\quad + (\rho c_p)_{eff}A_{cr}\frac{\partial T}{\partial t}dx
\end{aligned} \tag{7}$$

Dividing Eq. (7) through by

$$\begin{aligned}
&-\frac{1}{A_{cr}}\frac{\partial q}{\partial x} + (1-\phi)\,q'''(T) \\
&= \frac{hP\,(1-\phi)\,(T-T_a)}{A_{cr}} \\
&\quad + \frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}} \\
&\quad + \frac{\sigma\varepsilon P(T^4 - T_s^4)}{A_{cr}} \\
&\quad + (\rho c_p)_{eff}u\frac{\partial T}{\partial x} + (\rho c_p)_{eff}\frac{\partial T}{\partial t}
\end{aligned} \tag{8}$$

Eq. (8) can be written as

$$\begin{aligned}
&-\frac{1}{A_{cr}}\frac{\partial q}{\partial x} + (1-\phi)\,q'''(T) \\
&= \frac{hP\,(1-\phi)\,(T-T_a)}{A_{cr}} \\
&\quad + \frac{\rho_f c_{p,f} gK\beta\phi(T-T_a)^2}{v\delta} \\
&\quad + \frac{\sigma\varepsilon P(T^4 - T_s^4)}{A_{cr}} \\
&\quad + (\rho c_p)_{eff}u\frac{\partial T}{\partial x} + (\rho c_p)_{eff}\frac{\partial T}{\partial t}
\end{aligned} \tag{9}$$

The heat conduction rate through the solid portion of the fin is given by Fourier's law as given

$$q_s = -k_{eff}A_{cr}\frac{\partial T}{\partial x} \tag{10}$$

The radiation heat transfer rate in the porous medium is given as

$$q_p = -\frac{4\sigma A_{cr}\phi}{3\beta_R}\frac{\partial T^4}{\partial x} \tag{11}$$

Therefore, the total rate of heat transfer is given by

$$q = -k_{eff}A_{cr}\frac{\partial T}{\partial x} - \frac{4\sigma A_{cr}\phi}{3\beta_R}\frac{\partial T^4}{\partial x} \tag{12}$$

The introduction of Eq. (12) into Eq. (9) provides,

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(k_{eff}\frac{\partial T}{\partial x} + \frac{4\sigma\phi}{3\beta_R}\frac{\partial T^4}{\partial x}\right) + (1-\phi)\,\dot{q}'''(T) \\
&= \frac{\rho_f c_{p,f} gK\beta\phi(T-T_a)^2}{v\delta} \\
&\quad + \frac{hP\,(1-\phi)\,(T-T_\infty)}{A_{cr}} + \frac{\sigma\varepsilon P(T^4 - T_s^4)}{A_{cr}} \\
&\quad + (\rho c_p)_{eff}u\frac{\partial T}{\partial x} + (\rho c_p)_{eff}\frac{\partial T}{\partial t}
\end{aligned} \tag{13}$$

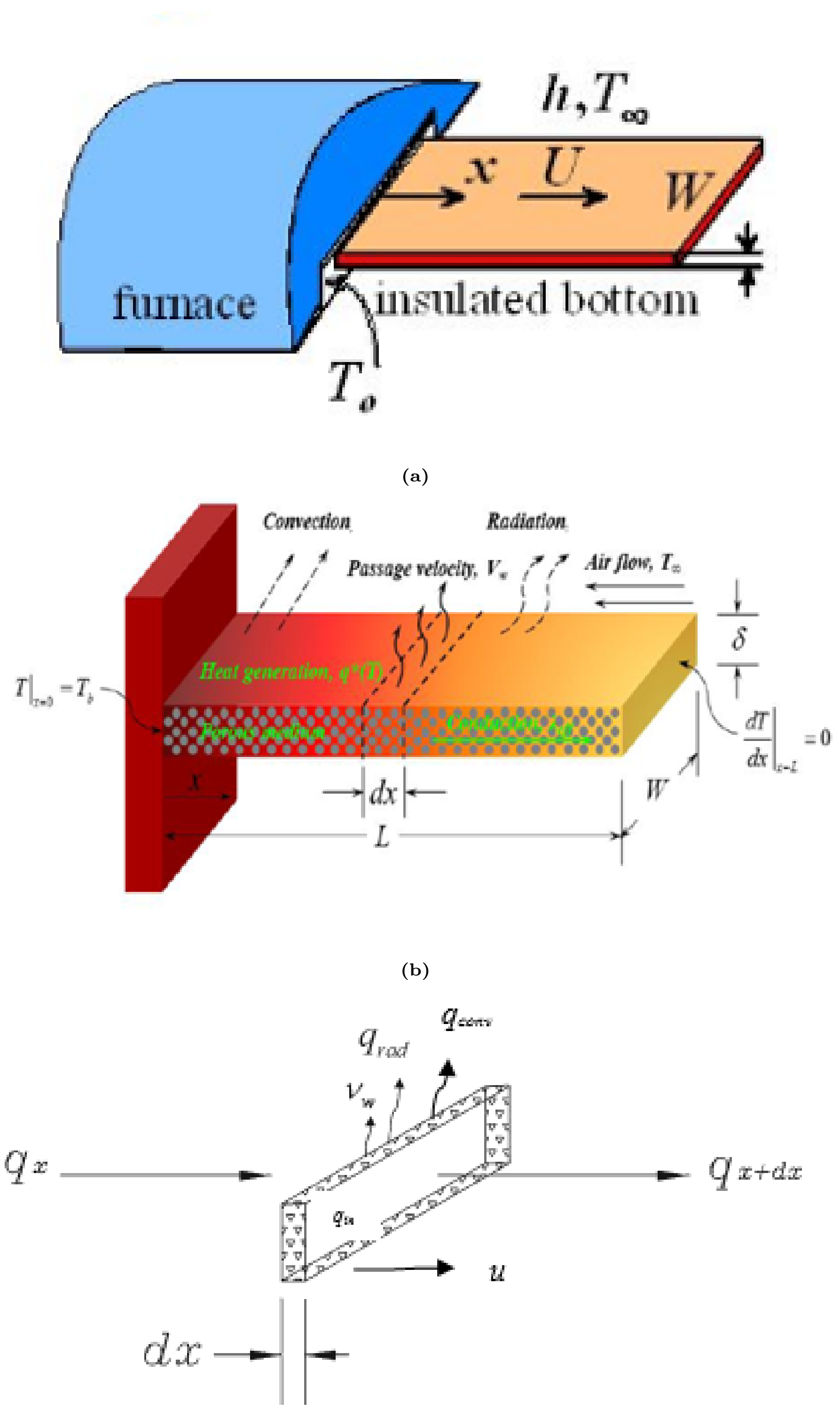


Fig. 1: (a) Schematic diagram of rolling and extrusion, (b) Schematic of a longitudinal moving porous fin with perfect thermal contact and insulated tip, (c) Thermal energy balance in the elemental strip.

Further simplification of Eq. (13) the governing differential equation for the fin becomes

$$\frac{\partial}{\partial x}\left(k_{eff}\frac{\partial T}{\partial x}\right)+\frac{4\sigma\phi}{3\beta_R}\frac{\partial}{\partial x}\left(\frac{\partial T^4}{\partial x}\right) - \frac{\rho_f c_{p,f} gK\beta\phi(T-T_a)^2}{v\delta} - \frac{hP(1-\phi)}{A_{cr}}(T-T_a) - \frac{\sigma\varepsilon P}{A_{cr}}(T^4-T_s^4) + (1-\phi)\,q'''(T) = (\rho c_p)_{eff}u\frac{\partial T}{\partial x} + (\rho c_p)_{eff}\frac{\partial T}{\partial t} \tag{14}$$

The temperature-dependent thermal conductivity and internal heat generation are respectively given by the linear expressions as

$$k_{eff} = (1-\phi)\,k_s + \phi k_f \tag{15}$$

where,

$$q'''(T) = q'''_o\,[1+\gamma(T-T_a)] \tag{16}$$

And

$$(\rho c_p)_{eff} = (1-\phi)\,(\rho c_p)_s + \phi(\rho c_p)_f \tag{17}$$

Therefore, the governing equation becomes

$$k_{eff}\frac{\partial^2 T}{\partial x^2}+\frac{4\sigma\phi}{3\beta_R}\frac{\partial}{\partial x}\left(\frac{\partial T^4}{\partial x}\right) - \frac{\rho_f c_{p,f} gK\beta\phi(T-T_a)^2}{v\delta} - \frac{h_b P(1-\phi)(T-T_a)}{A_{cr}} - \frac{\sigma\varepsilon_b P}{A_{cr}}\left(T^4-T_s^4\right) + (1-\phi)\,q'''_o\,[1 + \gamma(T-T_a)] = (\rho c_p)_{eff}\left[u\frac{\partial T}{\partial x}+\frac{\partial T}{\partial t}\right] \tag{18}$$

For the case when the temperature between the base and tip of the fin is small, the radiative term can be linearized. Using Roseland's approximation for the radiative term in the model, with the aid of Taylor series, expanding about

$$T^4 \cong T_a^4 + 4T_a^3\,(T-T_a) + 6T_a^2(T-T_a)^2 + 4T_a(T-T_a)^3 + \ldots \tag{19}$$

and ignoring the higher order components in Eq. (21a), we have

$$T^4 \cong 4T_a^3 T - 3T_a^4 \tag{20}$$

Therefore,

$$T^4 - T_a^4 \cong 4T_a^3\,(T-T_a) \tag{21}$$

Substituting Eq. (21) into the second term in the Eq. (18), we have

$$-\frac{4\sigma\phi}{3\beta_R}\frac{\partial T^4}{\partial x} = -\frac{4\sigma\phi}{3\beta_R}\frac{\partial(4T_a^3T-3T_a^3)}{\partial x} = -\frac{16\sigma\phi T_a^3}{3\beta_R}\frac{\partial T}{\partial x} \tag{22}$$

Substituting Eqs. (21c) and (21d) into Eq. (18), we have

$$k_{eff}\frac{\partial^2 T}{\partial x^2}+\frac{16\sigma\phi T_a^3}{3\beta_R}\frac{\partial^2 T}{\partial x^2} - \frac{\rho_f c_{p,f} gK\beta W\phi(T-T_a)^2}{vA_{cr}} - \frac{hP(1-\phi)}{A_{cr}}(T-T_a) - \frac{4\sigma\varepsilon PT_a^3}{A_{cr}}(T-T_a) + (1-\phi)\,q'''_o\,[1 + \gamma(T-T_a)] = (\rho c_p)_{eff}u\frac{\partial T}{\partial x} + (\rho c_p)_{eff}\frac{\partial T}{\partial t} \tag{23}$$

The initial condition is

$$T = T_0, \quad \text{when } t=0, \quad \text{for } 0 < x < L, \tag{24}$$

The boundary conditions for the fin with insulated tip are given as

$$T = T_b, \quad \text{at } x = 0, \quad \text{for } t>0, \tag{25a}$$

$$\frac{dT}{dx} = 0, \quad \text{at } x = L, \quad \text{for } t>0, \tag{25b}$$

(25)

Eq. (23) can be expressed as

$$\begin{aligned}&\left(k_{eff}+\frac{16\sigma\phi T_a^3}{3\beta_R}\right)\frac{\partial^2 T}{\partial x^2}\\&-\frac{\rho_f c_{p,f} gK\beta W\phi (T-T_a)^2}{vA_{cr}}\\&-\frac{hP(1-\phi)}{A_{cr}}(T-T_a)\\&-\frac{4\sigma\varepsilon PT_a^3}{A_{cr}}(T-T_a)+(1-\phi)q'''_o[1\\&+\gamma(T-T_a)]=(\rho c_p)_{eff}u_o\frac{\partial T}{\partial x}\\&+(\rho c_p)_{eff}\frac{\partial T}{\partial t}\end{aligned} \tag{26}$$

The temperature-dependent internal heat generation in the porous fin can be expressed as

$$\begin{aligned}q'''(T)=(1-\phi)q'''_o+(1-\phi)q'''_o\gamma T\\-(1-\phi)q'''_o\gamma T_a\end{aligned} \tag{27}$$

Therefore

$$\begin{aligned}&\left(k_{eff}+\frac{16\sigma\phi T_a^3}{3\beta_R}\right)\frac{\partial^2 T}{\partial x^2}\\&-\frac{\rho_f c_{p,f} gK\beta W\phi (T-T_a)^2}{vA_{cr}}\\&-\frac{hP(1-\phi)}{A_{cr}}(T-T_a)\\&-\frac{4\sigma\varepsilon PT_a^3}{A_{cr}}(T-T_a)+(1-\phi)q'''_o[1\\&+\gamma(T-T_a)]=(\rho c_p)_{eff}u\frac{\partial T}{\partial x}\\&+(\rho c_p)_{eff}\frac{\partial T}{\partial t}\end{aligned} \tag{28}$$

Collecting like terms, we have

$$\begin{aligned}&\left(k_{eff}+\frac{16\sigma\phi T_a^3}{3\beta_R}\right)\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi (T-T_a)^2}{vA_{cr}}-\begin{bmatrix}\frac{hP(1-\phi)}{A_{cr}}+\frac{4\sigma\varepsilon PT_a^3}{A_{cr}}\\-(\gamma(1-\phi)q'''_o)\end{bmatrix}(T-T_a)\\&+(1-\phi)q'''_o=(\rho c_p)_{eff}u\frac{\partial T}{\partial x}+(\rho c_p)_{eff}\frac{\partial T}{\partial t}\end{aligned} \tag{29}$$

Which can be written as

$$\begin{aligned}&\left(1+\frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}}\right)\frac{\partial^2 T}{\partial x^2}-\frac{\rho_f c_{p,f} gK\beta W\phi (T-T_a)^2}{vA_{cr}k_{eff}}-\begin{bmatrix}\frac{hP(1-\phi)}{A_{cr}k_{eff}}+\frac{4\sigma\varepsilon PT_a^3}{A_{cr}k_{eff}}\\-\frac{(\gamma(1-\phi)q'''_o)}{k_{eff}}\end{bmatrix}(T-T_a)\\&+\frac{(1-\phi)q'''_o}{k_{eff}}=\frac{(\rho c_p)_{eff}u}{k_{eff}}\frac{\partial T}{\partial x}+\frac{(\rho c_p)_{eff}}{k_{eff}}\frac{\partial T}{\partial t}\end{aligned} \tag{30}$$

Applying the following dimensionless variables

$$X=\frac{x}{L},\quad \theta=\frac{T-T_a}{T_b-T_a},\quad \tau=\frac{k_{eff}t}{(\rho c_p)_{eff}L^2} \tag{31}$$

Which implies that

$$\begin{aligned}x=XL,\quad &T\\=\theta(T_b-T_a)+T_a,\quad &T-T_a\\=\theta(T_b-T_a),\quad &t\\=\frac{\tau(\rho c_p)_{eff}L^2}{k_{eff}}\end{aligned} \tag{32}$$

When Eq. (32) is substituted into Eq. (30), we have

$$\begin{aligned}&\left(1+\frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}}\right)\frac{\partial^2[\theta(T_b-T_a)+T_a]}{\partial(XL)^2}-\frac{\rho_f c_{p,f} gK\beta W\phi[\theta(T_b-T_a)]^2}{vA_{cr}k_{eff}}\\&-\begin{bmatrix}\frac{hP(1-\phi)}{A_{cr}k_{eff}}+\frac{4\sigma\varepsilon PT_a^3}{A_{cr}k_{eff}}\\-\frac{(\gamma(1-\phi)q'''_o)}{k_{eff}}\end{bmatrix}\theta(T_b-T_a)+\frac{(1-\phi)q'''_o}{k_{eff}}\\&=\frac{(\rho c_p)_{eff}u}{k_{eff}}\frac{\partial[\theta(T_b-T_a)+T_a]}{\partial(XL)}+\frac{(\rho c_p)_{eff}}{k_{eff}}\frac{\partial[\theta(T_b-T_a)+T_a]}{\partial\left(\frac{(\rho c_p)_{eff}L^2\tau}{k_{eff}}\right)}\end{aligned} \tag{33}$$

Further simplification provides,

$$\begin{aligned}&\left(1+\frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}}\right)\frac{(T_b-T_a)}{L^2}\frac{\partial^2\theta}{\partial X^2}-(T_b-T_a)^2\frac{\rho_f c_{p,f} gK\beta W\phi\theta^2}{vA_{cr}k_{eff}}\\&-(T_b-T_a)\left[\frac{hP(1-\phi)}{A_{cr}k_{eff}}+\frac{4\sigma\varepsilon PT_a^3}{A_{cr}k_{eff}}-\frac{(\gamma(1-\phi)q'''_o)}{k_{eff}}\right]\theta\\&+\frac{(1-\phi)q'''_o}{k_{eff}}=\frac{(T_b-T_a)}{L}\frac{(\rho c_p)_{eff}u}{k_{eff}}\frac{\partial\theta}{\partial X}+\frac{(T_b-T_a)}{L^2}\frac{\partial\theta}{\partial\tau}\end{aligned} \tag{34}$$

Which gives

$$\begin{aligned}&\left(1+\frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}}\right)\frac{\partial^2\theta}{\partial X^2}-\frac{\rho_f c_{p,f} gK\beta W\phi L^2(T_b-T_a)\theta^2}{vA_{cr}k_{eff}}\\&-\left[\frac{hP(1-\phi)L^2}{A_{cr}k_{eff}}+\frac{4\sigma\varepsilon PT_a^3 L^2}{A_{cr}k_{eff}}-\frac{(\gamma(1-\phi)q'''_o)L^2}{k_{eff}}\right]\theta\\&+\frac{((1-\phi)q'''_o)L^2}{k_{eff}(T_b-T_a)}=\frac{(\rho c_p)_{eff}uL}{k_{eff}}\frac{\partial\theta}{\partial X}+\frac{\partial\theta}{\partial\tau}\end{aligned} \tag{35}$$

The above equation can be written as

$$(1+4R)\frac{\partial^2\theta}{\partial X^2}-S_h\theta^2-(Mc+Mr-Q_{\gamma e})\theta+Q_e=Pe_e\frac{\partial\theta}{\partial X}+\frac{\partial\theta}{\partial\tau} \tag{36}$$

where

$$Rd = \frac{16\sigma\phi T_a^3}{3\beta_R k_{eff}}, \quad Mc^2 = \frac{hP\left(1-\phi\right)L^2}{A_{cr}k_{eff}}, \quad Mr = \frac{4\sigma\varepsilon P T_a^3 L^2}{A_{cr}k_{eff}}, \quad Q_\gamma \frac{\gamma\left(1-\phi\right){q'''}_o L^2}{k_{eff}},$$
$$Q = \frac{\left(1-\phi\right){q'''}_o L^2}{k_{eff}\left(T_b - T_a\right)}, \quad Pe = \frac{\left(\rho c_p\right)_{eff} u_o L}{k_{eff}}, \quad S_h = \frac{\rho_f c_{p,f} g K \beta W \phi L^2 \left(T_b - T_a\right)}{v A_{cr} k_{eff}} \tag{37}$$

Eq. (36) is alternative written as

$$\frac{\partial^2\theta}{\partial X^2} - S_p\theta^2 - \left(Nc + Nr - Q_\gamma\right)\theta + Q = Pe_R\frac{\partial\theta}{\partial X} + \zeta\frac{\partial\theta}{\partial\tau} \tag{38}$$

where

$$Nc = \frac{Mc}{1+4R}, \quad Nr = \frac{Mr}{1+4R} \quad S_p = \frac{S_h}{1+4R}, \quad Q_\gamma = \frac{Q_{\gamma e}}{1+4R},$$
$$Q = \frac{Q_e}{(1+4R)}, \quad Pe_R = \frac{Pe_e}{(1+4R)}, \quad \zeta = \frac{1}{(1+4R)} \tag{39}$$

Also, the nondimensionalization for the initial and boundary conditions can be done by substituting Eq. (32) into Eqs. (24) and (25)

The initial condition is

$$\theta\left(T_b - T_a\right) + T_a = T_0 \quad \text{when} \frac{\tau\left(\rho c_p\right)_{eff} L^2}{k_{eff}} = 0, \quad \text{for } 0 < XL < L, \tag{40}$$

The boundary conditions for the fin with insulated tip are given as

$$\theta\left(T_b - T_a\right) + T_a = T_b \quad \text{at } XL = 0 \quad \text{for } \frac{\tau\left(\rho c_p\right)_{eff} L^2}{k_{eff}} > 0, \tag{41a}$$

$$\frac{\partial\left[\theta\left(T_b - T_a\right) + T_a\right]}{\partial(XL)} = 0 \quad \text{at } XL = L, \quad \text{for } \frac{\tau\left(\rho c_p\right)_{eff} L^2}{k_{eff}} > 0, \tag{41b}$$

(41)

Therefore, the adimensional initial condition is

$$\theta = \theta_0, \quad \text{when } \tau = 0, \quad \text{for } 0 < X < 1, \tag{42}$$

Also, the dimensionless boundary conditions are given as

$$\theta = 1, \quad \text{at } X = 0 \quad \text{for } \tau > 0, \tag{43a}$$

$$\frac{\partial\theta}{\partial X} = 0, \quad \text{at } X = 1, \quad \text{for } \tau > 0, \tag{43b}$$

(43)

3. Numerical Solution of the Nonlinear Transient thermal model

$$\left(\frac{\theta_{i+1}^{n+1} - 2\theta_i^{n+1} + \theta_{i-1}^{n+1} + \theta_{i+1}^{n} - 2\theta_i^{n} + \theta_{i-1}^{n}}{2\Delta^2 X}\right) - S_p(\theta_i^n)^2 - \left(Nc + Nr - Q_\gamma\right)\theta_i^n + Q = Pe_R\left(\frac{\theta_{i+1}^{n+1} - \theta_{i-1}^{n+1} + \theta_{i+1}^{n} - \theta_{i-1}^{n}}{4\Delta X}\right) + \zeta\left(\frac{\theta_i^{n+1} - \theta_i^n}{\Delta\tau}\right) \tag{44}$$

Which can be arranged as

$$\left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right)\theta_{i+1}^{n+1} - \left(\frac{2}{2\Delta^2 X} + \frac{\zeta}{\Delta\tau}\right)\theta_i^{n+1} + \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right)\theta_{i-1}^{n+1} + \left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right)\theta_{i+1}^{n} - \left(\frac{2}{2\Delta^2 X} - \frac{\zeta}{\Delta\tau} + \left(Nc + Nr - Q_\gamma\right)\right)\theta_i^n - S_p(\theta_i^n)^2 + \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right)\theta_{i-1}^{n} + Q = 0 \tag{45}$$

We can write Eq. (45) as

$$A\theta_{i+1}^{n+1} + B\theta_i^{n+1} + C\theta_{i-1}^{n+1} + D\theta_{i+1}^{n} + E\theta_i^n + F(\theta_i^n)^2 + G\theta_{i-1}^n + H = 0 \tag{46}$$

where

$$A = \left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right), B = -\left(\frac{2}{2\Delta^2 X} + \frac{\zeta}{\Delta\tau}\right),$$
$$C = \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right), D = \left(\frac{1}{2\Delta^2 X} - \frac{Pe_R}{4\Delta X}\right),$$
$$E = -\left(\frac{2}{2\Delta^2 X} - \frac{\zeta}{\Delta\tau} + \left(Nc + Nr - Q_\gamma\right)\right), F = -S_p,$$
$$G = \left(\frac{1}{2\Delta^2 X} + \frac{Pe_R}{4\Delta X}\right), H = Q,$$

The finite difference discretization of the initial condition is

$$\theta_i^o = 0 \ , \tag{47}$$

While FDM for boundary conditions become

$$\frac{\theta_1^n - \theta_{-1}^n}{2\Delta X} = 0 \quad \Rightarrow \quad \theta_1^n = \theta_{-1}^n \tag{48}$$

$$\theta_M^n = 1$$

4. Development of Exact Analytical Solution for the Linearized Model

In order to generate an exact analytical solution for the nonlinear model, we can linearize the nonlinear term in the porous term in Eq. (38).

$$\theta^2 = \theta_a^2 + 2\theta_a\left(\theta - \theta_a\right) = 2\theta_a\theta - \theta_a^2 \tag{49}$$

When Eq. (38) is substituted into Eq. (36), we have

$$\begin{aligned}\frac{\partial^2\theta}{\partial X^2} - S_p\left(2\theta_a\theta - \theta_a^2\right) - \left(Nc + Nr - Q_\gamma\right)\theta \\ + Q = Pe_R\frac{\partial\theta}{\partial X} + \zeta\frac{\partial\theta}{\partial\tau}\end{aligned} \tag{50}$$

Collecting the like terms provides

$$\begin{aligned}\frac{\partial^2\theta}{\partial X^2} - \left(Nc + Nr + 2\theta_a S_p - Q_\gamma\right)\theta \\ + Q + S_p\theta_a^2 = Pe_R\frac{\partial\theta}{\partial X} + \zeta\frac{\partial\theta}{\partial\tau}\end{aligned} \tag{51}$$

Applying Laplace transform to the transient thermal model in Eq. (51), we have

$$\begin{aligned}\frac{d^2\bar{\theta}}{dX^2} - \left(Nc + Nr + 2\theta_a S_p - Q_\gamma\right)\bar{\theta} \\ + \frac{Q + S_p\theta_a^2}{s} = Pe_R\frac{d\bar{\theta}}{dX} + \zeta\left(s\bar{\theta} - \theta_0\right)\end{aligned} \tag{52}$$

Collection of like terms provides

$$\begin{aligned}\frac{d^2\bar{\theta}}{dX^2} - Pe_R\frac{d\bar{\theta}}{dX} \\ - \left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)\bar{\theta} = \\ -\left(\frac{Q + S_p\theta_a^2}{s} + \zeta\theta_0\right)\end{aligned} \tag{53}$$

Also, the adimensional boundary conditions in the Laplace domain are

$$\theta = \frac{1}{s}, \quad \text{at } X = 0 \quad \text{for } s > 0, \tag{54a}$$

$$\frac{\partial\theta}{\partial X} = 0, \quad \text{at } X = 1, \quad \text{for } s > 0, \tag{54b}$$

(54)

On solving the above equation, one arrives at

$$\begin{aligned}\bar{\theta}(X,s) = Ae^{\left(\frac{Pe_R + \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)X} + Be^{\left(\frac{Pe_R - \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)X} \\ + \frac{\left(Q + S_p\theta_a^2 + s\zeta\theta_0\right)}{s\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}\end{aligned} \tag{55}$$

After applying the boundary conditions in Eq. (54a) and (54b), we have

$$A(s) = \frac{-\left[\left(\frac{Pe_R - \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)e^{\left(\frac{Pe_R - \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)}\right]\left[1 - \frac{\left(Q + S_p\theta_a^2 + s\zeta\theta_0\right)}{\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}\right]}{s\left[\begin{aligned}\left(\frac{Pe_R + \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)e^{\left(\frac{Pe_R + \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)} \\ -\left(\frac{Pe_R - \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)e^{\left(\frac{Pe_R - \sqrt{{Pe_R}^2 + 4\left(\zeta s + Nc + Nr + 2\theta_a S_p - Q_\gamma\right)}}{2}\right)}\end{aligned}\right]} \tag{56}$$

$$B(s)=\frac{\left[\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\right]\left[1-\frac{(Q+S_p\theta_a^2+s\zeta\theta_0)}{(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}\right]}{s\left[\begin{array}{l}\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\\-\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\end{array}\right]} \tag{57}$$

On substituting A and B into the solution in Eqs. (56) and (57), we have

$$\bar{\theta}(X,s)=\left[1-\frac{(Q+S_p\theta_a^2+s\zeta\theta_0)}{(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}\right]\left\{\frac{\left[\begin{array}{l}\left[\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)L}\right]e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)X}\\-\left[\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)L}\right]e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)X}\end{array}\right]}{s\left[\begin{array}{l}\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\\-\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\end{array}\right]}\right\}+\frac{(Q+S_p\theta_a^2+s\zeta\theta_0)}{(\zeta s+Nc+Nr+2\theta_a S_p-Q_\gamma)} \tag{58}$$

1

$$A=\frac{-\left[\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\right]\left[1-\frac{Q+S_p\theta_a^2}{(Nc+Nr+2\theta_a S_p-Q_\gamma)}\right]}{\left[\begin{array}{l}\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\\-\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\end{array}\right]} \tag{63}$$

$$B=\frac{\left[\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\right]\left[1-\frac{Q+S_p\theta_a^2}{(Nc+Nr+2\theta_a S_p-Q_\gamma)}\right]}{\left[\begin{array}{l}\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\\-\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{{Pe_R}^2+4(Nc+Nr+2\theta_a S_p-Q_\gamma)}}{2}\right)}\end{array}\right]} \tag{64}$$

On substituting Eqs. (63) and (64) into the solution in Eq. (65), we have

$$\theta(X)=\left[1-\frac{Q+S_p\theta_a^2}{(Nc+Nr+2\theta_aS_p-Q_\gamma)}\right]\left\{\frac{\left[\begin{array}{l}\left[\left(\frac{Pe_R+\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)L}\right]e^{\left(\frac{Pe_R-\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)X}\\-\left[\left(\frac{Pe_R-\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)L}\right]e^{\left(\frac{Pe_R+\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)X}\end{array}\right]}{\left[\begin{array}{l}\left(\frac{Pe_R+\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R+\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)}\\-\left(\frac{Pe_R-\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)e^{\left(\frac{Pe_R-\sqrt{Pe_R{}^2+4(Nc+Nr+2\theta_aS_p-Q_\gamma)}}{2}\right)}\end{array}\right]}\right\}+\frac{Q+S_p\theta_a^2}{(Nc+Nr+2\theta_aS_p-Q_\gamma)} \tag{65}$$

5. Results and Discussion

The simulated results and parametric studies on the passive device are presented in this section. The effect of each parameter of the thermal model on the thermal behaviour of the extended surface is investigated. The results are presented in various sub-sections for better analysis and understanding.

The Fig. 2a and 2b show the effect of Peclet number on the thermal response of the fin. While Fig. 2a illustrate the impact of low Peclet number on the thermal characteristics of the passive device, Fig. 2a shows the influence of very high Peclet number on the thermal performance of the extended surface. The results presented that the fin temperature increases with increases in the Peclet number. A low Peclet number depict a significant change in the fin temperature distribution from the fin base to the tip while for a very high Peclet number, there is marginal or negligible temperature difference between the fin base and its tip. This is because when the speed of the fin increases, the material moves faster and consequently, the material exposure time to the environment (the surrounding fluid) gets shortened and the rate of heat loss from the surface of the fin surface reduces, thereby causing increase in the fin temperature. Therefore, it could be stated that when cooling enhancement is needed, it is preferable to use a low Peclet number.

Figs. 3a-d and 4a-d displays the impacts of convective-conductive and radiative-conductive parameters under varying Peclet on the thermal behaviour of the fin. It is shown in the figures that the convective-conductive and radiative-conductive and Peclet numbers have significant effects on the heat transfer in the porous fin. The results depicted that as the convective-conductive and radiative-conductive parameters increase, the dimensionless temperature distribution in the fin decreases and consequently, the rate of heat transfer by the fin increases. The swift reduction in temperature is because as these parameters increase, more heat is lost from the fin because the heat transfer rate is enhanced, and more cooling of the fin occurs which shows a decrease in the temperature profile. Consequently, the fin thermal performance is increased. However, it is still shown that the fin temperature increases with increases in the Peclet number.

The effects of ambient and surface temperature on temperature distribution in the moving porous fin are shown in Figs. 5a-d and 6a-b. An increase in the Peclet number, ambient and surface temperatures resulted in increase the values of thermal distribution within the extended surface. This is expected because when the ambient and surface temperatures are increased, the rate convective and radiative heat loss form the surface of the fin will be reduced and more heat will be absorbed by the fin, thereby, the fin temperature history intensifies. Further simulations show that the temperature history increases with increasing time value. This is expected because with increasing heat transfer rate, the porous fin conducts more heat, thus temperature increases.

Figs. 8a-d analyze the effect of porosity parameters under varying Peclet on the thermal behaviour of the fin. It is shown in the figures that the porous term effect the fin ther-

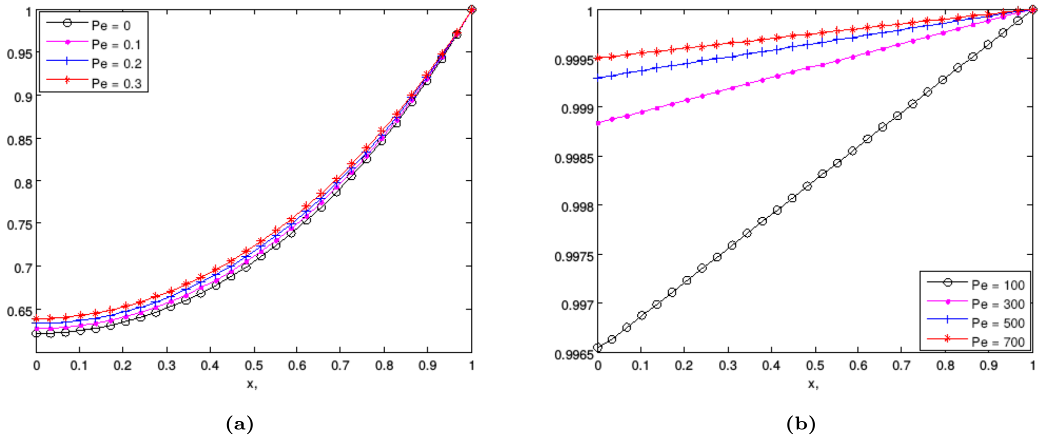


Fig. 2: (a) Effect of low Peclet number on the fin thermal behaviour when Nc=0.8 Nr=0.2, θ_a=0.1 θ_s=0.2, (b) Effect of high Peclet number on the fin thermal behaviour when Nc=0.8 Nr=0.2, θ_a=0.1 θ_s=0.2.

mal behaviour in a significant way. It is shown that when the porous term increases, there is decreases in the fin dimensionless temperature distribution which causes the rate of heat transfer by the fin to increase. The is due to increased porosity of the fin which enhances the rate at which heat is lost form the fin surface to the atmosphere. It is also shown in the figure that when the fin speed increases, which indicate an increase in the Peclet number, interactive time between the surface and surrounding drops. Therefore, it takes small time for the fin to release heat to the environment or being cooled by the surrounding fluid. Consequently, the temperature of the fin increases.

Fig. 9a-d depict the effect of internally generated heat on temperature distribution in the moving porous fin. From the plot, it is evident that an augmentation in the internally generated heat increases the magnitude of thermal distribution within the fin. This is expected because when the heat generation parameter is increased, more heat will be absorbed by the fin which consequently increases the fin temperature.

The results of the present study are compared and verified with the results of our past study using regular perturbation method (RPM) as presented in Tables 1 and 2. The excellent agreements between the two methods are shown as presented in the Tables

Tab. 1: Comparison of results of $\theta(X)$ when M = 0.8, Nr = 0.2 Sp= 0.0, Pe=0.

X	EXACT	RPM [62]	Difference
0.00	0.64805427	0.64805427	0.00000000
0.20	0.66105862	0.66105862	0.00000000
0.40	0.70059357	0.70059357	0.00000000
0.60	0.76824580	0.76824580	0.00000000
0.80	0.86673043	0.86673043	0.00000000
1.00	1.000000000	1.000000000	0.00000000

Tab. 2: Comparison of results of $\theta(X)$ when Nc = 0.40, Nr = 0.1, Sp= 0.0, Pe=0.0.

X	EXACT	RPM [62]	Difference
0.00	0.886818883	0.886818884	0.000000001
0.20	0.891256674	0.891256674	0.000000000
0.40	0.904614461	0.904614462	0.000000000
0.60	0.927025934	0.927025934	0.000000000
0.80	0.958715394	0.958715394	0.000000000
1.00	1.000000000	1.000000000	0.000000000

6. Conclusion

In this work, the effect of Peclet number on thermal behaviour of a porous fin is investigated. The results showed that the fin temperature increases with increase in the Peclet number. Therefore, it could be stated that when cooling enhancement is needed, it is preferable to use a low Peclet number. Also, under varying Peclet number, the fin thermal distribution decreases as porosity, conductive-convective

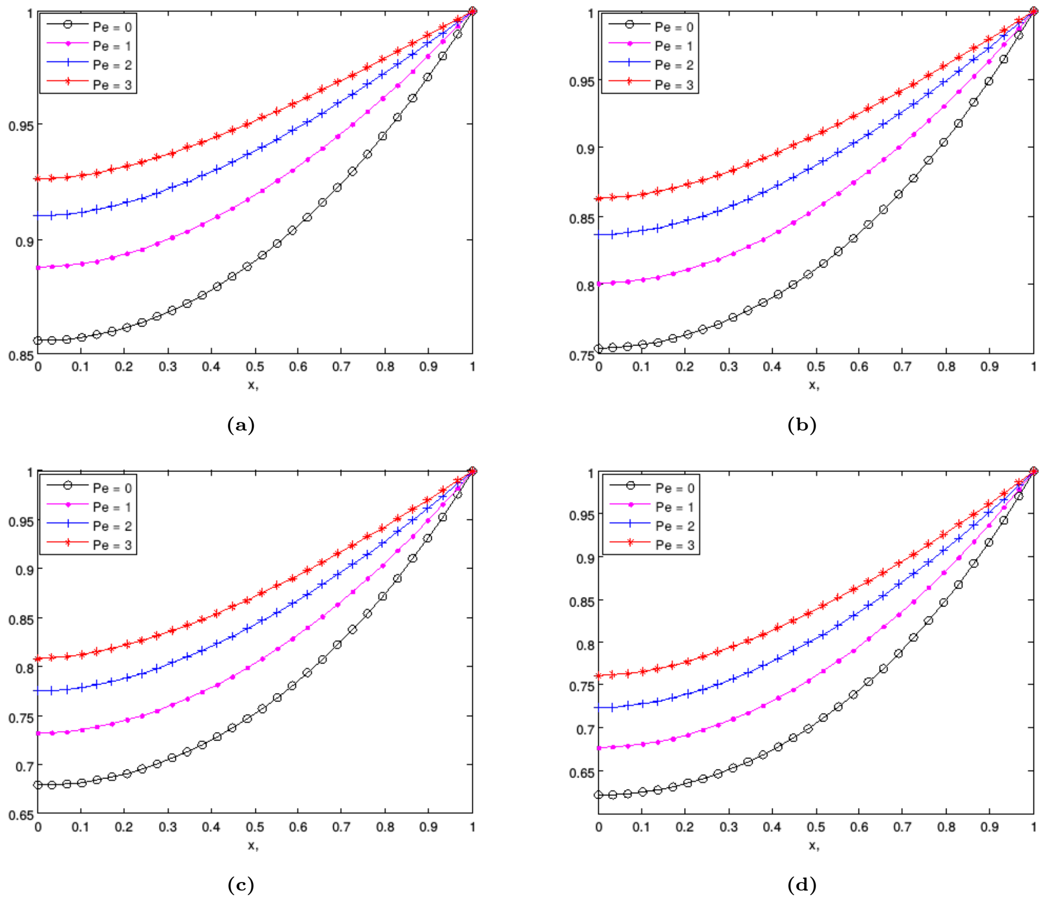


Fig. 3: (a) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, (b) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2.

and conductive-radiative terms increase. However, the temperature of the extended surface increases as internal heat generation, ambient and surface temperatures increase. It is hoped that the present study will help better understanding of the thermal problems in extended surfaces, especially on the effect of Peclet number on moving fin.

References

[1] Torabi, M., Yaghoobi, H., & Aziz, A. (2012). Analytical solution for convective–radiative continuously moving fin with temperature-dependent thermal conductivity. *International Journal of Thermophysics*, *33*, 924–941.

[2] Aziz, A. & Lopez, R.J. (2011). Convection-radiation from a continuously moving, variable thermal conductivity sheet or rod undergoing thermal processing. *International journal of thermal sciences*, *50*(8), 1523–1531.

[3] Aziz, A. & Khani, F. (2011). Convection–radiation from a continuously moving fin of variable thermal conductivity. *Journal of the Franklin Institute*, *348*(4), 640–651.

[4] Singh, S., Kumar, D., & Rai, K. (2013). Wavelet Collocation Solution for Convective-Radiative Continuously Moving

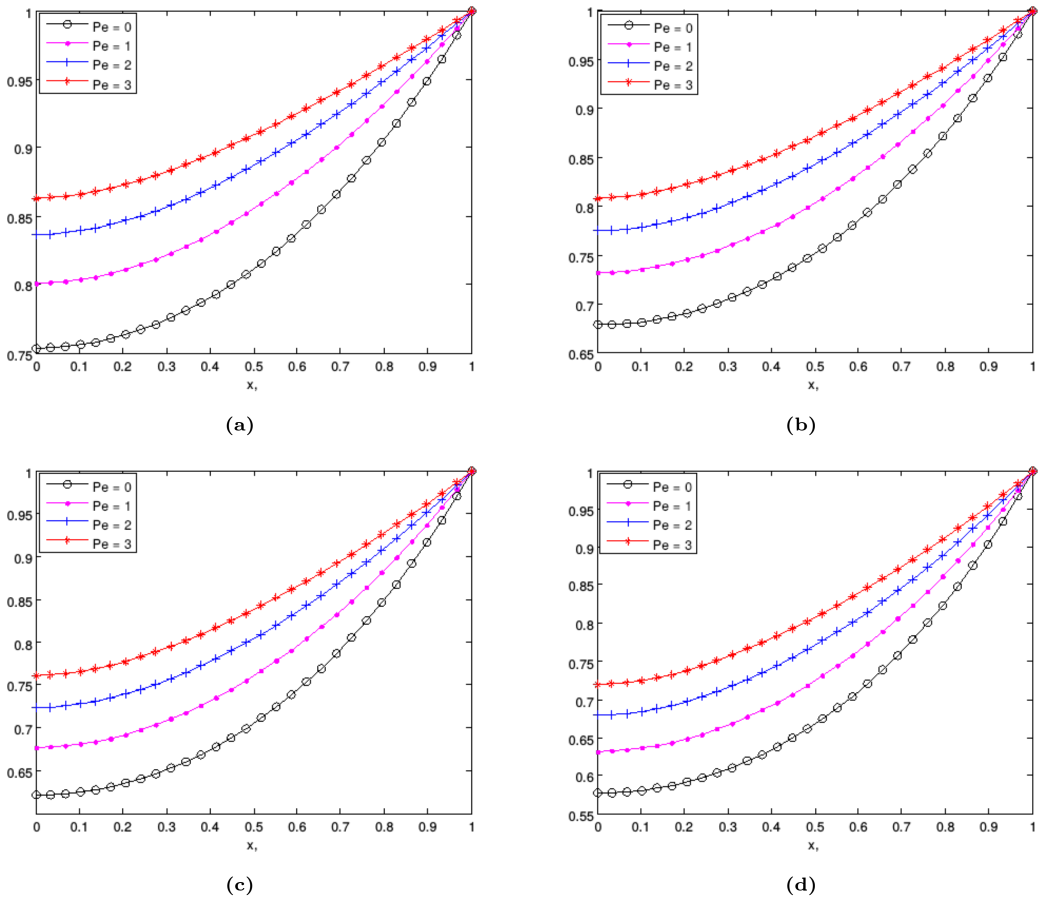


Fig. 4: (a) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, (b) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2.

Fin with Temperature-Dependent Thermal Conductivity. *International Journal of Engineering and Advanced Technology*, *2*(4), 10–16.

[5] Aziz, A. & Torabi, M. (2012). Convective-radiative fins with simultaneous variation of thermal conductivity, heat transfer coefficient, and surface emissivity with temperature. *Heat Transfer—Asian Research*, *41*(2), 99–113.

[6] Ma, J., Sun, Y., Li, B., & Chen, H. (2016). Spectral collocation method for radiative–conductive porous fin with temperature dependent properties. *Energy Conversion and Management*, *111*, 279–288.

[7] Sun, Y.S., Ma, J., & Li, B.W. (2015). Spectral collocation method for convective–radiative transfer of a moving rod with variable thermal conductivity. *International Journal of Thermal Sciences*, *90*, 187–196.

[8] Ravi Kanth, A. & Uday Kumar, N. (2013). Application of the Haar wavelet method on a continuously moving convective-radiative fin with variable thermal conductivity. *Heat Transfer—Asian Research*, *42*(4), 335–351.

[9] KANTH, A.S.V.R. & KUMAR, N.U. (2014). A haar wavelet study on convective-radiative fin under continuous motion with temperature-dependent thermal conductiv-

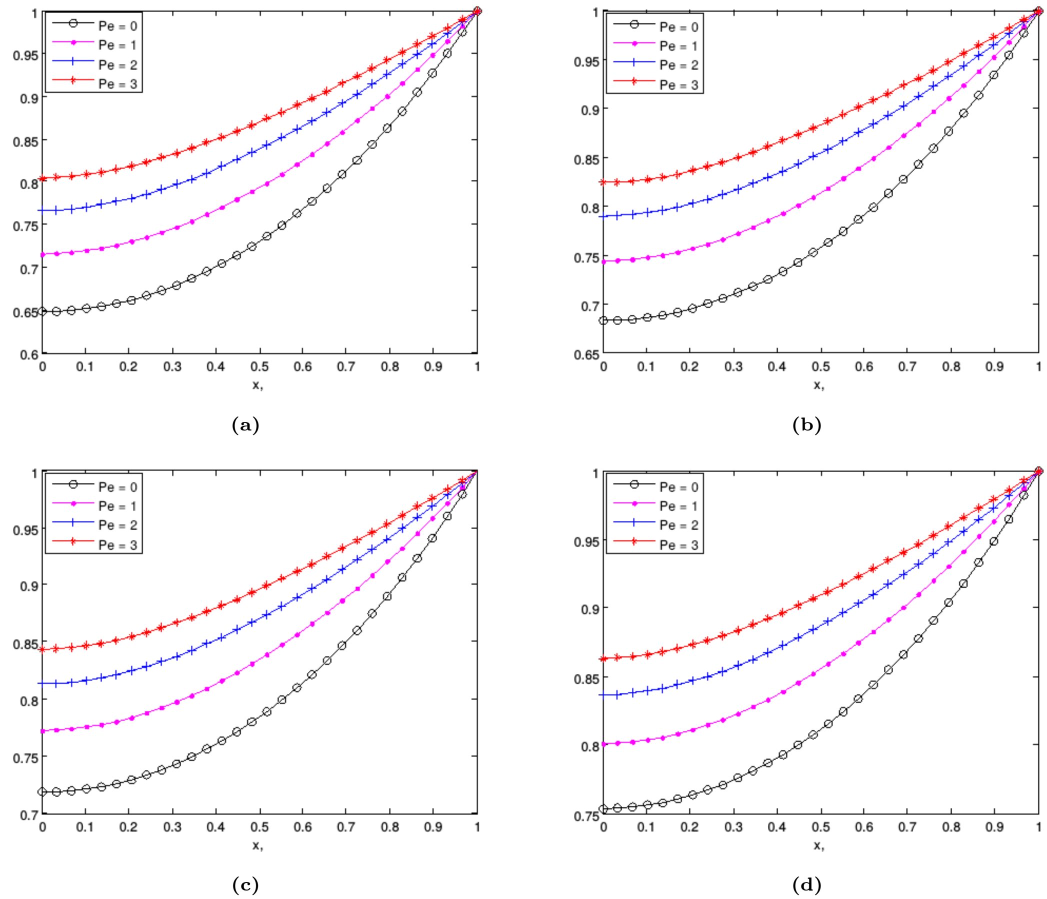


Fig. 5: (a) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, (b) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2.

ity. *Walailak Journal of Science and Technology (WJST)*, *11*(3), 211–224.

[10] Singla, R.K. & Das, R. (2014). Application of decomposition method and inverse prediction of parameters in a moving fin. *Energy Conversion and Management*, *84*, 268–281.

[11] Moradi, A. & Rafiee, R. (2013). Analytical solution to convection-radiation of a continuously moving fin with temperature-dependent thermal conductivity. *Thermal Science*, *17*(4), 1049–1060.

[12] Dogonchi, A. & Ganji, D. (2016). Convection–radiation heat transfer study of moving fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation. *Applied thermal engineering*, *103*, 705–712.

[13] Sun, Y. & Ma, J. (2015). Application of collocation spectral method to solve a convective-radiative longitudinal fin with temperature dependent internal heat generation, thermal conductivity and heat transfer coefficient. *Journal of Computational and Theoretical Nanoscience*, *12*(9), 2851–2860.

[14] Sobamowo, M., Yinusa, A., Salami, M., Osih, O., & Adesoye, B. (2021). Heat transfer analysis of a rectangular moving porous fin with temperature-dependent thermal

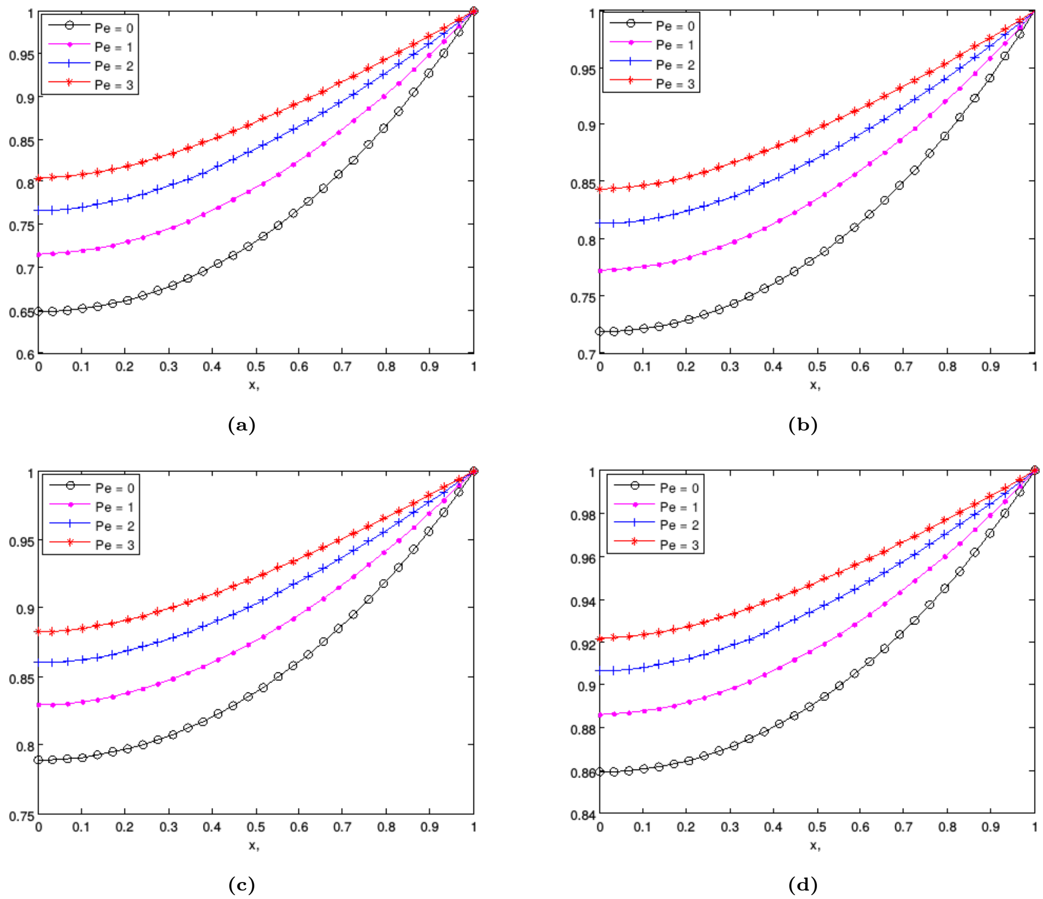


Fig. 6: (a) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, (b) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2.

conductivity and internal heat generation: Comparative and parametric studies. *Engineering Advances*, *1*(2), 50–66.

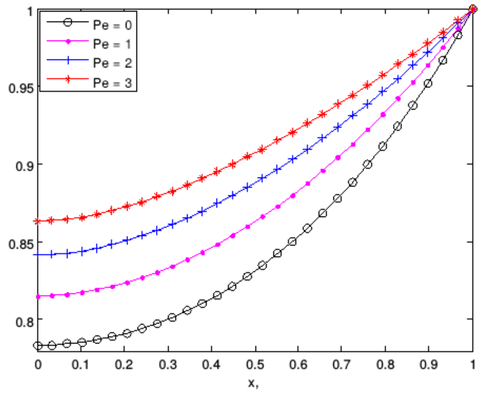
[15] Kim, T.Y., Kim, D.K., & Kim, S.J. (2008). Scroll heat sink: A novel heat sink with the moving fins inserted between the cooling fins. *International journal of heat and mass transfer*, *51*(13-14), 3267–3274.

[16] Jayaprakash, M., Alzahrani, H.A., Sowmya, G., Kumar, R.V., Malik, M., Alsaiari, A., & Prasannakumara, B. (2021). Thermal distribution through a moving longitudinal trapezoidal fin with variable temperature-dependent thermal properties using DTM-Pade approximant. *Case Studies in Thermal Engineering*, *28*, 101697.

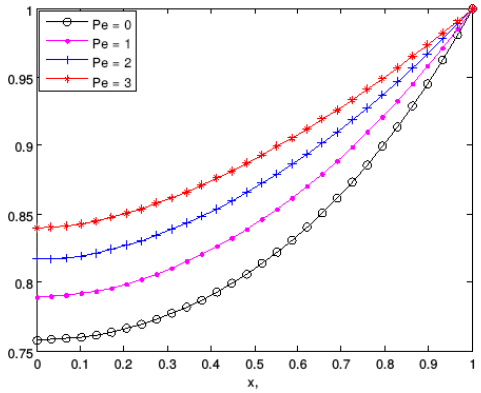
[17] Turkyilmazoglu, M. (2018). Heat transfer from moving exponential fins exposed to heat generation. *International Journal of Heat and Mass Transfer*, *116*, 346–351.

[18] Moradi, A. & Rafiee, R. (2013). Analytical solution to convection-radiation of a continuously moving fin with temperature-dependent thermal conductivity. *Thermal Science*, *17*(4), 1049–1060.

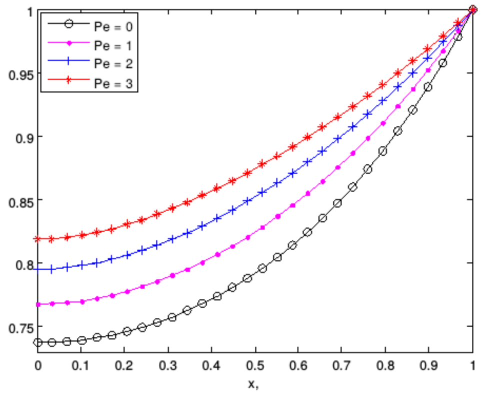
[19] Moradi, A., Fallah, A., Hayat, T., & Aldossary, O.M. (2014). On solution of natural convection and radiation heat transfer problem in a moving porous fin. *Arabian*



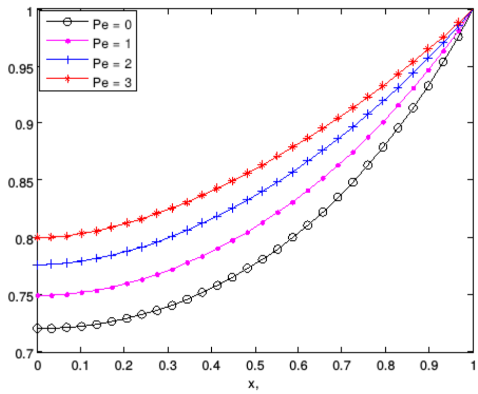
(a)



(b)



(c)



(d)

Fig. 7: (a) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, (b) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2.

Journal for Science and Engineering, *39*, 1303–1312.

[20] Ndlovu, P.L. (2020). Numerical analysis of transient heat transfer in radial porous moving fin with temperature dependent thermal properties. *Journal of Applied and Computational Mechanics*, *6*(1), 137–144.

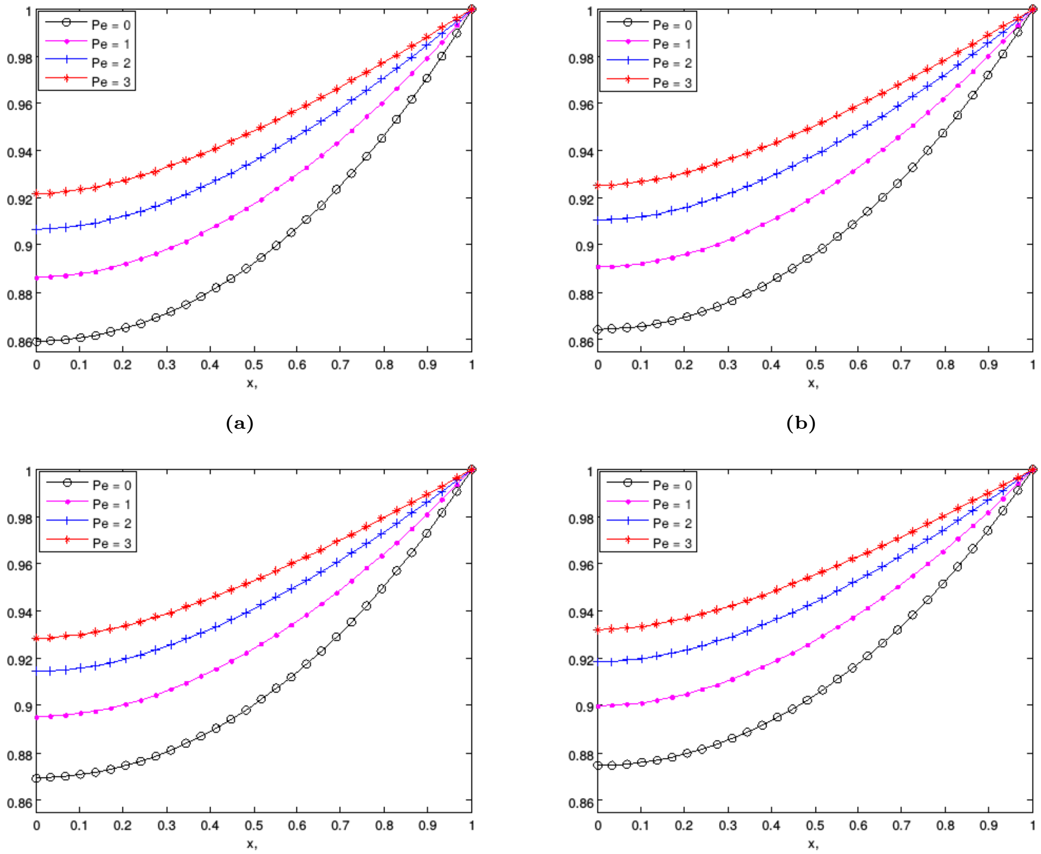
[21] Ndlovu, P.L. (2019). Analytical study of transient heat transfer in a triangular moving porous fin with temperature dependant thermal properties. In *Defect and Diffusion Forum*, volume 393, Trans Tech Publ, 31–46.

[22] Ndlovu, P. & Moitsheki, R. (2018). Thermal analysis of natural convection and radiation heat transfer in moving porous fins. *Frontiers in Heat and Mass Transfer (FHMT)*, *12*.

[23] Ndlovu, P.L. & Moitsheki, R.J. (2020). Steady state heat transfer analysis in a rectangular moving porous fin. *Propulsion and Power Research*, *9*(2), 188–196.

[24] Ndlovu, P.L. & Moitsheki, R.J. (2019). Analysis of transient heat transfer in radial moving fins with temperature-dependent thermal properties. *Journal of Thermal Analysis and Calorimetry*, *138*, 2913–2921.

[25] Ndlovu, P.L. & Moitsheki, R.J. (2019). Analysis of temperature distribution in radial moving fins with temperature depen-



(a) (b) (c) (d)

Fig. 8: (a) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, (b) Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2, Effect of Peclet number on the fin thermal behaviour when Nc=0.5 Nr=0.0, θ_a=0.3, θ_s=0.2.

dent thermal conductivity and heat transfer coefficient. *International Journal of Thermal Sciences*, *145*, 106015.

[26] Ndlovu, L.P. & Moitsheki, R.J. (2020). A study of transient heat transfer through a moving fin with temperature dependent thermal properties. In *Defect and Diffusion Forum*, volume 401, Trans Tech Publ, 1–13.

[27] Ndlovu, P.L. & Moitsheki, R.J. (2020). Analysis of a convective-radiative continuously moving fin with temperature-dependent thermal conductivity. *International Journal of Nonlinear Sciences and Numerical Simulation*, *21*(3-4), 379–388.

[28] Sowmya, G., Gireesha, B.J., & Madhu, M. (2020). Analysis of a fully wetted moving fin with temperature-dependent internal heat generation using the finite element method. *Heat Transfer*, *49*(4), 1939–1954.

[29] BJ, G. (2020). Analysis of thermal behavior of moving longitudinal porous fin wetted with water-based SWCNTs and MWCNTs. *Heat transfer*, *49*(4), 2044–2058.

[30] Sowmya, G. & Gireesha, B.J. (2022). Analysis of heat transfer through different profiled longitudinal porous fin by differential transformation method. *Heat Transfer*, *51*(2), 2165–2180.

[31] Gireesha, B., Sowmya, G., & Macha, M. (2019). Temperature distribution analysis in a fully wet moving radial porous fin by finite element method. *International Journal of Numerical Methods for Heat & Fluid Flow*, *32*(2), 453–468.

[32] Fallah Najafabadi, M., Talebi Rostami, H., Hosseinzadeh, K., & Domiri Ganji, D. (2021). Thermal analysis of a moving fin using the radial basis function approximation. *Heat transfer*, *50*(8), 7553–7567.

[33] Sobamowo, G., Yinusa, A.A., Popoola, O.P., & Waheed, M. (2021). Transient thermal analysis of convective-radiative moving fin under the influences of magnetic field and time-dependent boundary condition. *Computational Sciences and Engineering*, *1*(2), 139–152.

[34] Ma, J., Sun, Y., & Li, B. (2017). Spectral collocation method for transient thermal analysis of coupled conductive, convective and radiative heat transfer in the moving plate with temperature dependent properties and heat generation. *International Journal of Heat and Mass Transfer*, *114*, 469–482.

[35] Ma, J., Sun, Y., & Li, B. (2017). Simulation of combined conductive, convective and radiative heat transfer in moving irregular porous fins by spectral element method. *International journal of thermal sciences*, *118*, 475–487.

[36] Hosseinzadeh, S., Hosseinzadeh, K., Hasibi, A., & Ganji, D. (2022). Thermal analysis of moving porous fin wetted by hybrid nanofluid with trapezoidal, concave parabolic and convex cross sections. *Case Studies in Thermal Engineering*, *30*, 101757.

[37] Turkyilmazoglu, M. (2020). Expanding/-contracting fin of rectangular profile. *International Journal of Numerical Methods for Heat & Fluid Flow*, *31*(4), 1057–1068.

[38] Bhanja, D., Kundu, B., & Aziz, A. (2014). Enhancement of heat transfer from a continuously moving porous fin exposed in convective–radiative environment. *Energy conversion and management*, *88*, 842–853.

[39] Sun, Y.S. & Xu, J.L. (2015). Thermal performance of continuously moving radiative–convective fin of complex cross-section with multiple nonlinearities. *International Communications in Heat and Mass Transfer*, *63*, 23–34.

[40] Roy, P.K., Mallick, A., Mondal, H., & Sibanda, P. (2018). A modified decomposition solution of triangular moving fin with multiple variable thermal properties. *Arabian Journal for Science and Engineering*, *43*, 1485–1497.

[41] Sowmya, G., Sarris, I.E., Vishalakshi, C.S., Kumar, R.S.V., & Prasannakumara, B.C. (2021). Analysis of transient thermal distribution in a convective–radiative moving rod using two-dimensional differential transform method with multivariate pade approximant. *Symmetry*, *13*(10), 1793.

[42] Chen, H., Ma, J., & Liu, H. (2018). Least square spectral collocation method for nonlinear heat transfer in moving porous plate with convective and radiative boundary conditions. *International Journal of Thermal Sciences*, *132*, 335–343.

[43] Saedodin, S. & Barforoush, M.S.M. (2014). Comprehensive analytical study for convective–radiative continuously moving plates with multiple non-linearities. *Energy conversion and management*, *81*, 160–168.

[44] Ullah, S., Ali, A., & Din, Z. (2021). Heat transfer analysis in stretching/shrinking rectangular fin with convection and radiation. *Authorea Preprints*.

[45] Gireesha, B., Sowmya, G., & Nikitha, M. (2020). Numerical investigation of natural convection and radiation in a moving radial porous fin. *J Eng Manage*, *4*(1).

[46] Gireesha, B., Sowmya, G., & Nikitha, M. (2020). Numerical investigation of natural convection and radiation in a moving radial porous fin. *J Eng Manage*, *4*(1).

[47] Sobamowo, G. (2020). Finite element thermal analysis of a moving porous fin with temperature-variant thermal conductivity and internal heat generation. *Reports in Mechanical Engineering*, *1*(1), 110–127.

[48] Singh, S., Kumar, D., & Rai, K. (2014). Convective-radiative fin with temperature dependent thermal conductivity, heat transfer coefficient and wavelength dependent surface emissivity. *Propulsion and Power Research*, *3*(4), 207–221.

[49] Sobamowo, M., Salami, M., & Yinusa, A. (2022). Thermal Analysis of a Convective-Radiative Moving Porous Trapezoidal Fin with Variable Thermal Properties and Internal Heat Generation using Finite Element Method. *World Scientific News*, *163*, 139–157.

[50] Nabati, M., Taherifar, S., & Jalalvand, M. (2021). Sinc–Galerkin approach for thermal analysis of moving porous fin subject to nanoliquid flow with different shaped nanoparticles. *Mathematical Sciences*, 1–16.

[51] Gireesha, B., Sowmya, G., & Gorla, R.S.R. (2020). Nanoparticle shape effect on the thermal behaviour of moving longitudinal porous fin. *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*, *234*(3-4), 115–121.

[52] Gireesha, B.J., Sowmya, G., Khan, M.I., & Öztop, H.F. (2020). Flow of hybrid nanofluid across a permeable longitudinal moving fin along with thermal radiation and natural convection. *Computer methods and programs in biomedicine*, *185*, 105166.

[53] Turkyilmazoglu, M. (2017). Efficiency of the longitudinal fins of trapezoidal profile in motion. *Journal of Heat Transfer*, *139*(9).

[54] Sobamowo, M. & Waheed, M. (2020). Analytical investigation of heat transfer in a moving convective porous fin with temperature dependent thermal conductivity and internal heat generation. *World Scientific News*, (150), 1–21.

[55] Ndlovu, P.L. & Moitsheki, R.J. (2020). Steady state heat transfer analysis in a rectangular moving porous fin. *Propulsion and Power Research*, *9*(2), 188–196.

[56] OGUNTALA, G., Sobamowo, G., Yinusa, A., & Abd-Alhameed, R. (2020). Determination of Proper Fin Length of a Convective-Radiative Moving Fin of Functionally Graded Material Subjected to Lorentz Force. In *Defect and Diffusion Forum*, volume 401, Trans Tech Publ, 14–24.

[57] Gireesha, B.J., Sowmya, G., Khan, M.I., & Öztop, H.F. (2020). Flow of hybrid nanofluid across a permeable longitudinal moving fin along with thermal radiation and natural convection. *Computer methods and programs in biomedicine*, *185*, 105166.

[58] Sobamowo, G., Kamiyo, O., Salami, M., & Yinusa, A. (2019). Exploration of the effects of fin geometry and material properties on thermal performance of convective-radiative moving fins. *Engineering and Applied Science Letters*, *2*(3), 14–29.

[59] Sobamowo, M., Kamiyo, O., Salami, M., & Yinusa, A. (2020). Thermal assessment of a convective porous moving fins of different material properties using Laplace-variational iterative method. *World Scientific News*, *139*(2), 135–154.

[60] Simon, R.M., Stroot, M.T., & Weiss, G.H. (1972). Numerical inversion of Laplace transforms with application to percentage labeled mitoses experiments. *Computers and Biomedical Research*, *5*(6), 596–607.

[61] Lee, S., Chien, M., & Culham, W. (1984). Vertical single-well pulse testing of a three-layer stratified reservoir. In *SPE Annual technical conference and exhibition*, OnePetro.

[62] Sobamowo, M. & Oyediran, A. Effects of Thermogeometric parameter on the heat transfer rate in Straight fin wit variable thermal conductivity. *A Festschrift Celebrating*, *70*.

