Chattering-Free Second Order Robustness Sliding Mode Controller for Complex Interconnected Systems: A Moore-Penrose Inverse Technique

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Abstract. *In this paper, a novel second order sliding mode control approach is extended to a class of complex interconnected systems with mismatched interconnections and unknown perturbations. The key contribution of this paper is to cancel two common restrictions clogging the application of the variable structure control to complex interconnected systems: 1) all of state variables must be accessible; 2) the control input is affected by chattering problems. First, a novel reduced-order sliding mode estimator (ROSME) is proposed to estimate the unmeasurable variables. Second, based on Moore-Penrose inverse technique and ROSME tool, a new decentralized chattering-free second order robustness sliding mode controller (CSORSMC) is developed to force the system states to stay in a sliding surface from the initial period instance and attenuate the chattering phenomenon in the control signal. Besides, a novel appropriate linear matrix inequality (LMI) requirement is proved such that the plant in sliding mode is asymptotically stable. Lastly, a mathematical model including two subsystems is simulated which confirms the usefulness and advantages of proposed technique.*

Keywords: Sliding mode control, reduced-order sliding mode estimator, chattering-free, complex interconnected systems, without reaching phase.

1. Introduction

Variable structure control with sliding mode, also called sliding mode control (SMC), is a striking and robust control method for controlling of uncertain dynamical plants [1, 2]. Its advantages include the simple controller execution, insensitivity to the perturbations, and immense robustness to the uncertainties etc. [3–5]. Owing to these benefits, SMC is wonderfully employed for many practical control systems such as induction machines, the pitch angle of an aircraft, fuel cells, mechanical systems, solar photovoltaic energy systems, nuclear reactor, etc. [6–10]. Even good performance of sliding mode, in general, there are still two assignments that should be regarded for SMC design: This includes: 1) design a decentralized second order controller utilizing only output variables: in many existing researches, the state variables have to be accessible. This is worthless in practical control plants; 2) attenuate the undesirable high-frequency vibrations and consider the general systems: A new SMC strategy not only ensures the asymptotic stability of the complex interconnected systems but also reduces the chattering phenomenon in control signal by using the second order sliding mode technique.

For solving the decentralized output feedback control design problem in the above first task, many studies have been proposed in the papers [11–16]. In [11], a decentralized state feedback control law was sketched for the linear uncertain interconnected systems by using linear matrix inequality (LMI). In [12], an adaptive robust state feedback controller with a pretty simpler construction was suggested for a class of large-scale non-linear dynamical plants with non-linear interconnection terms and time-varying delayed state disturbances. Based on the backstepping approach and neural network technique, an adaptive neural network tracking control signal was established in [13] for interconnected nonlinear time-delay plants with external perturbations. Nevertheless, these researches have assumed that the variables of the plant are obtainable. This is invalid in the practical control systems. In order to resolve this weakness, the authors in the studies [14–16] have used the output feedback technique. In [14], an observer-based adaptive output feedback controller was constructed for a class of large-scale non-linear time delay plants by using Lyapunov-Krasovskii functional. In [15], a decentralized switched control strategy was investigated based on the K-filters and the backstepping design technique for a class of uncertain nonlinear interconnected plants. By utilizing the approximation ability of radial basis function neural networks, an observer-adaptive backstepping decentralized controller was developed in [16] for a class of powerfully interconnected nonlinear plants tolerating stochastic perturbations. Nevertheless, these works could not reduce the chattering influence in signal input. The chattering is one of undesirable impacts in applications of practical control (e.g., high heat losses in electrical power circuits, rebound in mechanical structures); it can lead to low control precision, abortive reduction or even humiliation of the plant performances being applied to. This also is the second task of our research.

For achieving the chattering mitigation in the second task, there are so many approaches to inhibit the negative effects of the chattering in the control plants such as [17–22]. By applying the backstepping-link technique, a second order sliding mode controller (SOSMC) was constructed in [17] for nonlinear constrained systems. In [18], a SOSMC was designed based on LMI technique for stabilizing the control of the uncertain plants with external perturbations, time-varying uncertainties, and nonlinearities. In [19], a second order sliding mode controller was built for the nonlinear plants by using a barrier Lyapunov function and the totaling a power integrator approach. By using this method, a SOSMC was investigated for the non linear plants with an asymmetric output restriction [20]. Recently, an adaptive second order controller was developed in [21] for the nonlinear system by means of the Lyapunov approach. In [22], an output feedback control signal was established to diminish the undesirable high-frequency vibrations for a class of uncertain switched plants by utilizing tanh function. Unfortunately, these studies have some solemn restrictions, where it is essential that the exogenous disturbances must be constrained by positive scalars and the full state information must be available. In addition, the existing works could not be applied for the stability of complex interconnected plants with the mismatched uncertainties in state matrix and interconnections.

Inspired by the above-mentioned analysis, in this paper, we challenge to address a novel decentralized chattering-free second order robustness sliding mode controller (CSORSMC) for a class of mismatched uncertain interconnected systems via the Moore-Penrose inverse technique. Firstly, a reduced-order sliding mode estimator (ROSME) is proposed to guess immeasurable variables of the plants. Secondly, a new CSORSMC is established by employing only output information and estimated state variables. Then, a creating LMI stipulation to ensure the complex interconnected systems with mismatched uncertainties in interconnections and state matrix in sliding mode is asymptotically stable. Finally, the theory will also be applied to two subsystems that revised from the study [22]. These numerical simulations demonstrate efficiency and feasibility of the theoretical results.

The remainder of this research is arranged as follows: The plant's mathematical model, preliminaries and a regular form are considered in Section II. The main achievements of the paper are represented in Section III, which comprises a new ROSME, the system's asymptotic stability analysis and the design of a decentralized CSORSMC. The practicability of the proposed method is explained in Section IV with simulation example by MATLAB software. To end this work, a conclusion is made in Section V.

2. Model description of the system

In this paper, a general mathematical model of mismatched uncertain interconnected schemes is portrayed as

$$\begin{aligned} \dot{x}_i(t) &= \left[\bar{A}_{ii} + \Delta\bar{A}_{ii}(t)\right] x_i(t) + \bar{B}_i\left[u_i(t) + \psi_i(x_i(t),t)\right] \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{L} \left[\bar{H}_{ij} + \Delta\bar{H}_{ij}(t)\right] x_j(t), \\ y_i(t) &= \bar{C}_i x_i(t), \end{aligned} \tag{1}$$

where $x_i(t) \in R^{n_i}, u_i(t) \in R^{m_i}, y_i(t) \in R^{p_i}$ are respectively the system states, the control signals, and the the i^{th} subsystem's output. The indexes n_i, m_i, p_i are the number of the system states, control inputs, and output channels, respectively. The symbols $j, L, \sum$ are respectively the indexes of the interconnection subsystem, the subsystems number and the sum of the interconnection subsystems. The matrices $\bar{A}_{ii}, \bar{B}_i, \bar{H}_{ij}, \bar{C}_i$ are constant matrices with suitable dimensions. The sign $\psi_i(x_i(t),t)$ is the exogenous disturbance of the plant. The matrix $\Delta\bar{A}_{ii}(t)x_i(t)$ shows the mismatched parameter uncertainty of the plant for each isolated subsystem. The terms $\sum\limits_{j=1,j\neq i}^{L} \bar{H}_{ij}x_j(t)$ and $\sum\limits_{j=1,j\neq i}^{L} \Delta\bar{H}_{ij}(t)x_j(t)$ indicate the mismatched uncertain interconnections in the dynamic equations of the i^{th} subsystem.

With the purpose of the CSORSMC design for the plant, the following assumptions will be introduced:

Assumption 1: The amount of control signals is smaller than or equal to the amount of the output channels, that is, $m_i \leq p_i$ and $p_i < n_i$. The input matrices and are full rank and $\text{rank}(\bar{C}_i\bar{B}_i) = m_i$.

Assumption 2: The pairs $(\bar{A}_{ii}, \bar{B}_i)$ and $(\bar{A}_{ii}, \bar{C}_i)$ are entirely controllable and observable, respectively.

Assumption 3: For the state matrix of each isolated subsystem, the mismatched parameter uncertainty $\Delta\bar{A}_{ii}(t)$ has to gratify $\bar{D}_{ii}\Xi_i(x_i(t),t)\bar{E}_{ii}$, where $\Xi_i(x_i(t),t)$ is unidentified but constrained by $\|\Xi_i(x_i(t),t)\| \leq 1$ for all $(x_i,t) \in R^{n_i} \times R$.

Assumption 4: The mismatched uncertain interconnection $\Delta\bar{H}_{ij}(t)$ must fulfill $\bar{F}_{ij}\Xi_{ij}(x_j(t),t)\bar{G}_{ij}$, where $\Xi_{ij}(x_j(t),t)$ is unidentified but constrained by $\|\Xi_{ij}(x_j(t),t)\| \leq 1$ for all $(x_j,t) \in R^{n_i} \times R$.

In order to create a novel attenuated-chattering single phase output feedback control algorithm, a single-phase sliding manifold function is defined as:

$$\sigma_i(y_i(t),t) = \dot{s}_i(y_i(t),t) + \tilde{X}_i s_i(y_i(t),t) \ , \tag{2}$$

where $s_i(y_i(t),t) = \bar{s}_i(y_i,t) - \bar{s}_i(y_i,0)\exp(-\upsilon_i t)$, $\dot{s}_i(y_i(t),t)$ is the time derivative of the term $s_i(y_i(t),t)$, $\bar{s}_i(y_i,t) = T_i x_i = P_i\bar{C}_i x_i = P_i y_i$, $\tilde{X}_i \in R^{m_i\times m_i}$ is any diagonal matrix and υ_i is positive constant. In addition, T_i is switching matrix and P_i is chosen matrix such that the expression $T_i = P_i\bar{C}_i$ is solvable.

To get the regular form of the mismatched uncertain interconnected systems (1), the contributed results in the paper [23, 24] will be utilized. There are symmetric matrices exist M_i and N_i satisfy the two following the LMIs:

$$\begin{aligned} &R_i M_i R_i + B_i N_i \bar{B}_i^T > 0 \ , \\ &\bar{B}_i^{\perp T}\left(\bar{A}_i R_i M_i R_i + R_i N_i R_i \bar{A}_i^T\right)\bar{B}_i^{\perp} < 0, \end{aligned} \tag{3}$$

where R_i and R_j are $n_i \times n_i$ symmetric matrices such that:

$$\begin{aligned} &R_i = I_i if \bar{B}_i^{\perp T}\bar{D}_{ii} = 0, \\ &R_i = I_i - \bar{E}_{ii}^g \bar{E}_{ii} if \bar{B}_i^{\perp T}\bar{D}_{ii} \neq 0, \\ &\text{and} \\ &R_j = I_j if \bar{B}_i^{\perp T}\bar{F}_{ij} = 0, \\ &R_j = I_j - \bar{G}_{ij}^g \bar{G}_{ij} if \bar{B}_i^{\perp T}\bar{F}_{ij} \neq 0, \end{aligned} \tag{4}$$

where the Moore-Penrose inverse of the matrices $\bar{E}_{ii}$ and $\bar{G}_{ij}$ are $\bar{E}_{ii}^{g}$ and $\bar{G}_{ij}^{g}$ respectively, and basis of the null space of the matrix $\bar{B}_i$ is $\bar{B}_i^{\perp}$.

The sliding matrix T_i is parameterized by $T_i = \Gamma_i \bar{B}_i^T S_i^{-1}$ where Γ_i is any $m_i \times m_i$ nonsingular matrix and $S_i = R_i M_i R_i + B_i N_i \bar{B}_i^T$. Now, in order to achieve the regular type of the original systems (1), the transformation matrix Π_i is suggested as:

$$\Pi_i = \begin{bmatrix} \bar{B}_i^{\perp T} \\ \Gamma_i \bar{B}_i^T S_i^{-1} \end{bmatrix}$$
and
$$\begin{bmatrix} \xi_i \\ \sigma_i \end{bmatrix} = \Pi_i x_i, \tag{5}$$

where the variable ξ_i is immeasurable and the switching manifold function σ_i is measurable. The transformation matrix's inverse Π_i is $\Pi_i^{-1} = [S_i \bar{B}_i^{\perp} (\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp})^{-1} \bar{B}_i (T_i \bar{B}_i)^{-1}]$. Now, by using the transformation matrix (5), we can get:

$$\begin{aligned}
\dot{\xi}_i &= \left[\tilde{A}_{ii11} + \Delta\tilde{A}_{ii11}\right]\xi_i + \left[\tilde{A}_{ii12} + \Delta\tilde{A}_{ii12}\right]\sigma_i \\
&+ \sum_{\substack{j=1 \\ j\neq i}}^{L} \left[\tilde{H}_{ij11} + \Delta\tilde{H}_{ij11}\right]\xi_j + \sum_{\substack{j=1 \\ j\neq i}}^{L} \left[\tilde{H}_{ij12} + \Delta\tilde{H}_{ij12}\right]\sigma_j, \\
\dot{\sigma}_i &= \left[\tilde{A}_{ii21} + \Delta\tilde{A}_{ii21}\right]\xi_i + \left[\tilde{A}_{ii22} + \Delta\tilde{A}_{ii22}\right]\sigma_i \\
&+ (T_i \bar{B}_i)\left[u_i(t) + \psi_i(x_i(t), t)\right] \\
&+ \sum_{\substack{j=1 \\ j\neq i}}^{L} \left[\tilde{H}_{ij21} + \Delta\tilde{H}_{ij21}\right]\xi_j + \sum_{\substack{j=1 \\ j\neq i}}^{L} \left[\tilde{H}_{ij22} + \Delta\tilde{H}_{ij22}\right]\sigma_j,
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
\tilde{A}_{ii11} + \Delta\tilde{A}_{ii11} &= \bar{B}_i^{\perp T}\left[\bar{A}_i + \bar{D}_i \Xi_i \bar{E}_i\right] S_i \bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\right)^{-1}, \\
\tilde{A}_{ii12} + \Delta\tilde{A}_{ii12} &= \bar{B}_i^{\perp T}\left[\bar{A}_i + \bar{D}_i \Xi_i \bar{E}_i\right] \bar{B}_i (T_i \bar{B}_i)^{-1}, \\
\tilde{A}_{ii21} + \Delta\tilde{A}_{ii21} &= \Gamma_i \bar{B}_i^T S_i^{-1}\left[\bar{A}_i + \bar{D}_i \Xi_i \bar{E}_i\right] S_i \bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\right)^{-1}, \\
\tilde{A}_{ii22} + \Delta\tilde{A}_{ii22} &= \Gamma_i B_i^T S_i^{-1}\left[\bar{A}_i + \bar{D}_i \Xi_i \bar{E}_i\right] \bar{B}_i (T_i \bar{B}_i)^{-1}, \\
\tilde{H}_{ij11} + \Delta\tilde{H}_{ij11} &= \bar{B}_i^{\perp T}\left[\bar{H}_{ij} + \bar{F}_{ij} \Xi_{ij} \bar{G}_{ij}\right] S_i \bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\right)^{-1}, \\
\tilde{H}_{ij12} + \Delta\tilde{H}_{ij12} &= \bar{B}_i^{\perp T}\left[\bar{H}_{ij} + \bar{F}_{ij} \Xi_{ij} \bar{G}_{ij}\right] \bar{B}_i (T_i \bar{B}_i)^{-1}, \\
\tilde{H}_{ij21} + \Delta\tilde{H}_{ij21} &= T_i\left[\bar{H}_{ij} + \bar{F}_{ij} \Xi_{ij} \bar{G}_{ij}\right] S_i \bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\right)^{-1}, \\
\tilde{H}_{ij22} + \Delta\tilde{H}_{ij22} &= T_i\left[\bar{H}_{ij} + \bar{F}_{ij} \Xi_{ij} \bar{G}_{ij}\right] \bar{B}_i (T_i \bar{B}_i)^{-1}, \\
\xi_i &= \bar{B}_i^{\perp T} x_i.
\end{aligned} \tag{7}$$

We generally consider the mismatching condition case of the interconnected uncertain systems (1). According to the possessions of the Moore-Penrose inverse technique and attainments in paper [25], we may simply acquire:

$$\begin{aligned}
\Delta\tilde{A}_{ii11} &= \bar{B}_i^{\perp T} \bar{D}_i \Xi_i \bar{E}_i S_i \bar{B}_i^{\perp} (\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp})^{-1} = 0, \\
\Delta\tilde{A}_{ii21} &= \Gamma_i \bar{B}_i^T S_i^{-1} \bar{D}_i \Xi_i \bar{E}_i S_i \bar{B}_i^{\perp} (\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp})^{-1} = 0, \\
\Delta\tilde{H}_{ij11} &= \bar{B}_i^{\perp T} \bar{F}_{ij} \Xi_{ij} \bar{G}_{ij} S_i \bar{B}_i^{\perp} \left(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\right)^{-1} = 0, \\
\Delta\tilde{H}_{ij21} &= T_i \bar{F}_{ij} \Xi_{ij} \bar{G}_{ij} S_i \bar{B}_i^{\perp} \left(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\right)^{-1} = 0.
\end{aligned} \tag{8}$$

By combining the Eqs. (6) and (8), the regular system may be characterized by the following:

$$\begin{aligned}
\dot{\xi}_i &= \tilde{A}_{ii11}\xi_i + \left[\tilde{A}_{ii12} + \Delta\tilde{A}_{ii12}\right]\sigma_i \\
&+ \sum_{\substack{j=1 \\ j\neq i}}^{L} \left\{\tilde{H}_{ij11}\xi_j + \left[\tilde{H}_{ij12} + \Delta\tilde{H}_{ij12}\right]\sigma_j\right\}, \\
\dot{\sigma}_i &= \tilde{A}_{ii21}\xi_i + \left[\tilde{A}_{ii22} + \Delta\tilde{A}_{ii22}\right]\sigma_i \\
&+ \left(T_i \bar{B}_i\right)\left[u_i(t) + \psi_i(x_i(t), t)\right] \\
&+ \sum_{\substack{j=1 \\ j\neq i}}^{L} \left\{\tilde{H}_{ij21}\xi_j + \left[\tilde{H}_{ij22} + \Delta\tilde{H}_{ij22}\right]\sigma_j\right\}.
\end{aligned} \tag{9}$$

3. Main results

3.1. Establishment of a novel reduced order sliding mode estimator for systems:

In this part, we will suggest a new ROSME which supports to construct a CSORSMC for the uncertain interconnected plants (1). The ROSME is suggested to guess the immeasurable states of the plants.

$$\dot{\hat{\xi}}_i(t) = \tilde{A}_{ii11}\hat{\xi}_i(t) + \tilde{A}_{ii12}\sigma_i(t)\ , \tag{10}$$

where $\hat{\xi}_i(t)$ is the estimate of $\xi_i(t)$. The block diagram of i^{th} subsystem being structured by ROSME (10) is showed in Figure 1. We define an error change between the the estimate variables and real states as $\tilde{\xi}_i(t) = \hat{\xi}_i(t) - \xi_i(t)$. Next, by merging the first of Eq. (6), results (8), and Eq.

(10) lead to the estimator error dynamics as

$$\dot{\tilde{\xi}}_i(t) = \tilde{A}_{ii11}\tilde{\xi}_i(t) - \Delta\tilde{A}_{ii12}\sigma_i - \sum_{\substack{j=1\\ j\neq i}}^{L}\left\{\tilde{H}_{ij11}\xi_j + \left[\tilde{H}_{ij12} + \Delta\tilde{H}_{ij12}\right]\sigma_j\right\}. \tag{11}$$

Based on the Eq. (7) and the property

$$\sum_{\substack{j=1\\ j\neq i}}^{L}\left\{\tilde{H}_{ij11}\xi_j + \left[\tilde{H}_{ij12} + \Delta\tilde{H}_{ij12}\right]\sigma_j\right\} = \sum_{\substack{j=1\\ j\neq i}}^{L}\left\{\tilde{H}_{ji11}\xi_i + \left[\tilde{H}_{ji12} + \Delta\tilde{H}_{ji12}\right]\sigma_i\right\}, \tag{12}$$

we have

$$\dot{\tilde{\xi}}_i(t) = \tilde{A}_{ii11}\tilde{\xi}_i(t) - \bar{B}_i^{\perp T}\bar{D}_i\Xi_i(x_i,t)\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\sigma_i - \sum_{\substack{j=1\\ j\neq i}}^{L}\left\{\tilde{H}_{ji11}\xi_i + \left[\tilde{H}_{ji12} + \bar{B}_j^{\perp T}\bar{F}_{ji}\Xi_{ji}\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right]\sigma_i\right\}. \tag{13}$$

To determine the upper limit of the estimator

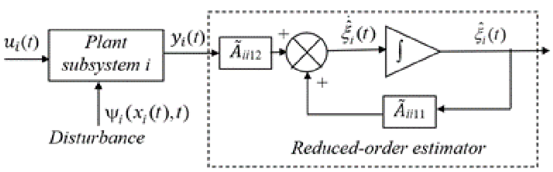

Fig. 1: The block diagram of the plant with ROSME (10).

error, the following theorem will be proposed as

Theorem 1. The norm of approximation error $\left\|\tilde{\xi}_i(t)\right\|$ in the observer error equation (13) is limited by $\varpi_i(t)$ for $t \geq 0$ The term $\varpi_i(t)$ is the result of

$$\dot{\varpi}_i(t) = \mu_i(t)\varpi_i(t) + \gamma_i\left[\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\|\left\|\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\| \times \left\|\sigma_i(t)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left\|\tilde{H}_{ji11}\right\|\left\|\hat{\xi}_i(t)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left[\left\|\tilde{H}_{ji12}\right\| + \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\|\left\|\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right]\left\|\sigma_i\right\|\right] \tag{14}$$

where $\mu_i(t) = \lambda_{\max_i} + \gamma_i \sum_{j=1, j\neq i}^{L}\left\|\tilde{H}_{ji11}\right\| < 0$, $\lambda_{\max_i}$ is a maximum eigenvalue of the matrix $\tilde{A}_{ii11}$, γ_i is positive constant, $\hat{\xi}_i(t)$ is the unmeasurable state estimator defined in the Eq. (10) and an initial condition of the approximation error $\varpi_i(0) \geq \gamma_i\left\|\tilde{\xi}_i(0)\right\|$.

Proof of Theorem 1. Following the recent study [25], the matrix $\tilde{A}_{ii11} = \bar{B}_i^{\perp T}\bar{A}_iS_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}$ is stable matrix. Thus, we have $\left\|\exp\left(\tilde{A}_{ii11}t\right)\right\| \leq \gamma_i\exp\left(\lambda_{\max_i}t\right)$, where γ_i is positive scalars, and solving (13) to yield

$$\begin{aligned}
\left\|\tilde{\xi}_i(t)\right\| &\leq \left\|\exp\left(\tilde{A}_{ii11}t\right)\right\|\left\|\tilde{\xi}_i(0)\right\| + \int_0^t\left\|\exp\left[\tilde{A}_{ii11}(t-\tau)\right]\right\| \\
&\times\left\{\left\|\bar{B}_i^{\perp T}\bar{D}_i\Xi_i(x_i,t)\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|\left\|\sigma_i(\tau)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left[\left\|\tilde{H}_{ji11}\right\|\left\|\xi_i(\tau)\right\| + \left(\left\|\tilde{H}_{ji12}\right\| + \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\Xi_{ji}\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right)\left\|\sigma_i(\tau)\right\|\right]\right\}d\tau, \\
&\leq \gamma_i\left\|\tilde{\xi}_i(0)\right\|\exp\left(\lambda_{\max_i}t\right) + \int_0^t\gamma_i\exp\left[\lambda_{\max_i}(t-\tau)\right] \\
&\times\left\{\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\|\left\|\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|\left\|\sigma_i(\tau)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left[\left\|\tilde{H}_{ji11}\right\|\left\|\xi_i(\tau)\right\| + \left(\left\|\tilde{H}_{ji12}\right\| + \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\|\left\|\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right)\left\|\sigma_i(\tau)\right\|\right]\right\}d\tau.
\end{aligned} \tag{15}$$

Now, the inequation (15) are multiplied by $\exp(-\lambda_{\max_i}t)$ for both sides and we get

$$\begin{aligned}
&\left\|\tilde{\xi}_i(t)\right\|\exp\left(-\lambda_{\max_i}t\right) \\
&\leq \gamma_i\left\|\tilde{\xi}_i(0)\right\| + \int_0^t\gamma_i\exp\left(-\lambda_{\max_i}\tau\right)\left\{\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\|\left\|\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\| \times\left\|\sigma_i(\tau)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left[\left\|\tilde{H}_{ji11}\right\|\left(\left\|\hat{\xi}_i(\tau)\right\| + \left\|\tilde{\xi}_i(\tau)\right\|\right) + \left(\left\|\tilde{H}_{ji12}\right\| + \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\|\left\|\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right)\left\|\sigma_i(\tau)\right\|\right]\right\}d\tau \\
&\leq \gamma_i\left\|\tilde{\xi}_i(0)\right\| + \int_0^t\gamma_i\exp\left(-\lambda_{\max_i}\tau\right)\sum_{\substack{j=1\\ j\neq i}}^{L}\left\|\tilde{H}_{ji11}\right\|\left\|\tilde{\xi}_i(\tau)\right\| \\
&+ \int_0^t\gamma_i\exp\left(-\lambda_{\max_i}\tau\right)\left[\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\|\left\|\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|\left\|\sigma_i(\tau)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left\|\tilde{H}_{ji11}\right\|\left\|\hat{\xi}_i(\tau)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L}\left(\left\|\tilde{H}_{ji12}\right\| + \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\| \times\left\|\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right)\left\|\sigma_i(\tau)\right\|\right]d\tau.
\end{aligned} \tag{16}$$

Shift $\exp(-\lambda_{\max_i}t)$ to the right-hand side index of above equation and use the Lemma in study

[26], it tracks that

$$\begin{aligned}
&\left\|\tilde{\xi}_i(t)\right\| \le \gamma_i \left\|\tilde{\xi}_i(0)\right\| \exp\left[\left(\lambda_{\max_i} + \gamma_i \sum_{\substack{j=1\\ j\neq i}}^{L} \left\|\tilde{H}_{ji11}\right\|\right) t\right]\\
&+ \int_0^t \gamma_i \exp\left[\left(\lambda_{\max_i} + \gamma_i \sum_{\substack{j=1\\ j\neq i}}^{L} \left\|\tilde{H}_{ji11}\right\|\right) (t-\tau)\right]\\
&\times \Bigg[\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\| \left\|\bar{E}_i\bar{B}_i(T_i\bar{B}_i)^{-1}\right\| \|\sigma_i(\tau)\| + \sum_{\substack{j=1\\ j\neq i}}^{L} \left\|\tilde{H}_{ji11}\right\| \left\|\hat{\xi}_i(\tau)\right\|\\
&+ \sum_{\substack{j=1\\ j\neq i}}^{L} \left(\left\|\tilde{H}_{ji12}\right\| + \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\| \left\|\bar{G}_{ji}\bar{B}_j(T_j\bar{B}_j)^{-1}\right\|\right) \|\sigma_i(\tau)\|\Bigg] d\tau,\\
&\le \varpi_i(0) \exp\left[\left(\lambda_{\max_i} + \gamma_i \sum_{\substack{j=1\\ j\neq i}}^{L} \left\|\tilde{H}_{ji11}\right\|\right) t\right]\\
&+ \int_0^t \gamma_i \exp\left[\left(\lambda_{\max_i} + \gamma_i \sum_{\substack{j=1\\ j\neq i}}^{L} \left\|\tilde{H}_{ji11}\right\|\right) (t-\tau)\right]\\
&\times \Big[\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\| \left\|\bar{E}_i\bar{B}_i(T_i\bar{B}_i)^{-1}\right\| \|\sigma_i(\tau)\|\\
&+ \sum_{j=1, j\neq i}^{L} \left\|\tilde{H}_{ji11}\right\| \left\|\hat{\xi}_i(\tau)\right\| + \sum_{\substack{j=1\\ j\neq i}}^{L} \Big(\left\|\tilde{H}_{ji12}\right\|\\
&+ \left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\| \left\|\bar{G}_{ji}\bar{B}_j(T_j\bar{B}_j)^{-1}\right\|\Big) \|\sigma_i(\tau)\|\Big] d\tau = \varpi_i(t),
\end{aligned} \tag{17}$$

where $\varpi_i(t)$ gratifies (14). From now, we can conclude that $\left\|\tilde{\xi}_i(t)\right\| \le \varpi_i(t)$ for all time. Accordingly, the proof of Theorem 1 is completed.

Now, we are in situation to derive necessary conditions by LMI such that the closed-loop system (1) is asymptotically stable in the sliding mode.

3.2. Stability analysis in single-phase sliding mode

In this section, the asymptotic stability of the overall systems in the sliding mode will be demonstrated by using the well-known LMI approach, Schur complement formula, and the Lyapunov function.

Theorem 2. Regard the mismatched uncertain interconnected systems (1) with assumptions 1-4 and the switching manifold surface $\sigma_i(y_i(t), t) = 0$. If there exist symmetric matrix

$$\begin{bmatrix} \Theta_i & \tilde{E}_i^T & \widehat{Q}_i\tilde{D}_i \\ \tilde{E}_i & -\varphi_{i1}^{-1} I_i & 0 \\ \tilde{D}_i^T\widehat{Q}_i & 0 & -\varphi_{i1} I_i \end{bmatrix} < 0 \tag{18}$$

where $\Theta_i = \tilde{A}_{ii11}^T\widehat{Q}_i + \widehat{Q}_i\tilde{A}_{ii11} + \sum_{j=1,j\neq i}^{L} \left[\tilde{H}_{ji11}^T\widehat{Q}_i + \widehat{Q}_i\tilde{H}_{ji11}\right.$ the scalar $\varphi_{i1} > 0$, and $\widehat{Q}_i \in R^{(n_i-m_i)\times(n_i-m_i)}$ is any positive definite matrix, then the overall plants (6) with the subsequent $(n_i - m_i)$ reduced-order dynamics is asymptotically stable in the sliding mode.

Proof of Theorem 2. By using the switching manifold surface $\sigma_i(y_i(t), t) = 0$, the sliding motion is offered by the following motion dynamics:

$$\dot{\xi}_i = \left[\tilde{A}_{ii11} + \tilde{D}_i\Xi_i\tilde{E}_i\right]\xi_i + \sum_{\substack{j=1\\ j\neq i}}^{L} \left[\tilde{H}_{ij11} + \tilde{F}_{ij}\Xi_{ij}\tilde{G}_{ij}\right]\xi_j, \tag{19}$$

where $\tilde{D}_i = f\bar{B}_i^{\perp T}\bar{D}_i, \tilde{E}_i = \bar{E}_iS_i\bar{B}_i^{\perp}(\bar{B}_i^{\perp \mathbf{T}}S_i\bar{B}_i^{\perp})^{-1}$, $\tilde{F}_{ij} = \bar{B}_i^{\perp T}\bar{F}_{ij}$, $\tilde{G}_{ij} = \bar{G}_{ij}S_i\bar{B}_i^{\perp}(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp})^{-1}$. Because $\sum_{j=1,j\neq i}^{L} \left[\tilde{H}_{ij11} + \tilde{F}_{ij}\Xi_{ij}\tilde{G}_{ij}\right]\xi_j = \sum_{j=1,j\neq i}^{L} \left[\tilde{H}_{ji11} + \tilde{F}_{ji}\Xi_{ji}\tilde{G}_{ji}\right]\xi_i$, the sliding motion (19) can be represented by:

$$\dot{\xi}_i = \left[\tilde{A}_{ii11} + \tilde{D}_i\Xi_i\tilde{E}_i\right]\xi_i + \sum_{\substack{j=1\\ j\neq i}}^{L} \left[\tilde{H}_{ji11} + \tilde{F}_{ji}\Xi_{ji}\tilde{G}_{ji}\right]\xi_i. \tag{20}$$

Now, by applying the Lyapunov function to above sliding motion dynamics, we have

$$V = \sum_{i=1}^{L} \xi_i^T\widehat{Q}_i\xi_i. \tag{21}$$

By differentiating V with respect time and combining with motion dynamics (20), we achieve

$$\dot{V}=\sum_{i=1}^{L}\left\{\xi_i^T\left[\tilde{A}_{ii11}^T\widehat{Q}_i+\widehat{Q}_i\tilde{A}_{ii11}+\tilde{E}_i^T\Xi_i^T(x_i,t)\tilde{D}_i^T\widehat{Q}_i+\widehat{Q}_i\tilde{D}_i\Xi_i(x_i,t)\tilde{E}_i\right]\xi_i\right\}+\sum_{i=1}^{L}\sum_{\substack{j=1\\j\neq i}}^{L}\xi_i^T\left[\tilde{H}_{ji11}^T\widehat{Q}_i+\widehat{Q}_i\tilde{H}_{ji11}+\tilde{G}_{ji}^T\Xi_{ji}^T\tilde{F}_{ji}^T\widehat{Q}_i+\widehat{Q}_i\tilde{F}_{ji}\Xi_{ji}\tilde{G}_{ji}\right]\xi_i. \tag{22}$$

Applying Lemma 1 in paper [27] to the Eq. (22), we obtain:

$$\dot{V}\le\sum_{i=1}^{L}\left\{\xi_i^T\left[\tilde{A}_{ii11}^T\widehat{Q}_i+\widehat{Q}_i\tilde{A}_{ii11}+\varphi_{i1}^{-1}\widehat{Q}_i\tilde{D}_i\tilde{D}_i^T\widehat{Q}_i+\varphi_{i1}\tilde{E}_i^T\tilde{E}_i+\sum_{\substack{j=1\\j\neq i}}^{L}\left(\tilde{H}_{ji11}^T\widehat{Q}_i+\widehat{Q}_i\tilde{H}_{ji11}+\varphi_{i2}^{-1}\widehat{Q}_i\tilde{F}_{ji}\tilde{F}_{ji}^T\widehat{Q}_i+\varphi_{i2}\tilde{G}_{ji}^T\tilde{G}_{ji}\right)\right]\xi_i\right\}. \tag{23}$$

By using Schur complement formula [28] to above equality, we get:

$$\tilde{A}_{ii11}^T\widehat{Q}_i+\widehat{Q}_i\tilde{A}_{ii11}+\sum_{\substack{j=1\\j\neq i}}^{L}\left[\tilde{H}_{ji11}^T\widehat{Q}_i+\widehat{Q}_i\tilde{H}_{ji11}+\varphi_{i2}^{-1}\widehat{Q}_i\tilde{F}_{ji}\tilde{F}_{ji}^T\widehat{Q}_i+\varphi_{i2}\tilde{G}_{ji}^T\tilde{G}_{ji}\right]+\varphi_{i1}\tilde{E}_i^T\tilde{E}_i+\varphi_{i1}^{-1}\widehat{Q}_i\tilde{D}_i\tilde{D}_i^T\widehat{Q}_i<0. \tag{24}$$

Based on the Eqs. (23) and (24), we can conclude $\dot{V}<0$, which further indicates that the dynamics of motion (19) is asymptotically stable. The proof of Theorem 2 is finished.

In order to continue a novel attenuated-chattering single-phase output feedback control algorithm which uses second order sliding mode control method, we will show it in following section.

3.3. Design a CSORSMC for reducing the chattering phenomenon

In the above section, Theorem 2 has been established for estimating error upper bound of the ROSME. Now, by using this theorem, we will suggest a decentralized CSORSMC to keep the system's state trajectory moving along switching manifold surface from the zero-reaching time. Firstly, by differentiating the switching manifold function $\sigma_i(y_i(t),t)$ with respect time, we have

$$\dot{s}_i=T_i\bar{A}_iS_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\xi_i(t)+T_i\bar{A}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\sigma_i+\left(T_i\bar{B}_i\right)u_i(t)+\sum_{\substack{j=1\\j\neq i}}^{L}\left[T_i\bar{H}_{ij}S_j\bar{B}_j^{\perp}\left(\bar{B}_j^{\perp T}S_j\bar{B}_j^{\perp}\right)^{-1}\times\xi_j(t)+T_i\bar{H}_{ij}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\sigma_j(t)\right]+\vartheta_i(t)+\upsilon_iP_iy_i(0)\exp(-\upsilon_it), \tag{25}$$

where

$$\vartheta_i(t)=T_i\Delta\bar{A}_i(t)S_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\xi_i(t)+T_i\Delta\bar{A}_i(t)\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\sigma_i+\left(T_i\bar{B}_i\right)\psi_i(x_i(t),t)+\sum_{\substack{j=1\\j\neq i}}^{L}\left[T_i\Delta\bar{H}_{ij}(t)S_j\bar{B}_j^{\perp}\left(\bar{B}_j^{\perp T}S_j\bar{B}_j^{\perp}\right)^{-1}\xi_j(t)+T_i\Delta\bar{H}_{ij}(t)\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\sigma_j(t)\right]. \tag{26}$$

The key idea of the second order sliding mode is to execute the second order derivative of the sliding variable $\ddot{s}_i(y_i(t),t)$ rather than the first derivative as in conventional sliding mode. The switching variable in the second order derivative is calculated as

$$\ddot{s}_i=T_i\bar{A}_iS_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\dot{\xi}_i+T_i\bar{A}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\dot{\sigma}_i(t)+\left(T_i\bar{B}_i\right)\dot{u}_i(t)+\sum_{\substack{j=1\\j\neq i}}^{L}\left[T_i\bar{H}_{ij}S_j\bar{B}_j^{\perp}\left(\bar{B}_j^{\perp T}S_j\bar{B}_j^{\perp}\right)^{-1}\dot{\xi}_j+T_i\bar{H}_{ij}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\dot{\sigma}_j(t)\right]+\dot{\vartheta}_i-\upsilon_i^2P_iy_i(0)\exp(-\upsilon_it). \tag{27}$$

The sliding function and its derivative are respectively showed as

$$\sigma_i(t)=T_i\bar{A}_iS_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\xi_i(t)+T_i\bar{A}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\sigma_i(t)+\sum_{\substack{j=1\\j\neq i}}^{L}\left[T_i\bar{H}_{ij}S_j\bar{B}_j^{\perp}\left(\bar{B}_j^{\perp \mathbf{T}}S_j\bar{B}_j^{\perp}\right)^{-1}\xi_j(t)+T_i\bar{H}_{ij}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\sigma_j(t)\right]+\vartheta_i(t)+\left(T_i\bar{B}_i\right)u_i(t)+\upsilon_iP_iy_i(0)\exp(-\upsilon_it)+\tilde{X}_i\left[\bar{s}_i(y_i,t)-\bar{s}_i(y_i,0)\exp(-\upsilon_it)\right], \tag{28}$$

and

$$\begin{aligned}\dot{\sigma}_i(t) &= T_i\bar{A}_iS_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\dot{\xi}_i(t)\\&+T_i\bar{A}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\dot{\sigma}_i(t)\\&+\sum_{\substack{j=1\\j\neq i}}^{L}\Big[T_i\bar{H}_{ij}S_j\bar{B}_j^{\perp}\left(\bar{B}_j^{\perp T}S_j\bar{B}_j^{\perp}\right)^{-1}\dot{\xi}_j(t)\\&+T_i\bar{H}_{ij}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\dot{\sigma}_j(t)\Big]+\dot{\vartheta}_i(t)\\&+\left(T_i\bar{B}_i\right)\dot{u}_i(t)+\tilde{X}_i\dot{\bar{s}}_i(y_i,t)+\left[\tilde{X}_i\upsilon_i-\upsilon_i^2\right]\\&\times\bar{s}_i(y_i,0)\exp(-\upsilon_it).\end{aligned}\tag{29}$$

Secondly, an unknown external disturbance $\dot{\vartheta}_i(t)$ is surmised to be constrained and to satisfy the following requirement:

$$\left\|\dot{\vartheta}_i(t)\right\|\leq\sum_{k=0}^{k}\left[\beta_{ki}\left\|y_i\right\|\left(\left\|x_i\right\|\right)^k\right]\tag{30}$$

where $\beta_{ki}, i = 1,2...,k$ is unknown positive scalar and k is the disturbance order. The positive integer r is found by the designer in conformity with the information about the order of the distractions. For example, if the perturbation order is 2, then $\left\|\dot{\vartheta}_i(t)\right\| \leq \beta_{0i}\left\|y_i\right\| + \beta_{1i}\left\|y_i\right\|\left(\left\|x_i\right\|\right) + \beta_{2i}\left\|y_i\right\|\left(\left\|x_i\right\|\right)^2$. By substituting (5) into (30) and using $\left\|\xi_i(t)\right\| \leq \left\|\hat{\xi}_i(t)\right\| + \varpi_i(t)$, the inequality (30) can be written as

$$\begin{aligned}\left\|\dot{\vartheta}_i(t)\right\|&\leq\sum_{k=0}^{k}\Big[\beta_{ki}\left\|y_i\right\|\Big(\left\|S_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\right\|\\&\times\left(\left\|\hat{\xi}_i(t)\right\|+\varpi_i(t)\right)+\left\|\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|\left\|\sigma_i\right\|\Big)^k\Big].\end{aligned}\tag{31}$$

Now, a CSORSMC is designed based on the attained consequences in Theorem 1 and 2 for dropping the chattering phenomenon in control signal and stabilizing the mismatched uncertain interconnected plants (1). This is main attainment of this research.

To impose the state variables of the closed-loop plants (6) upon the indicated sliding manifold (2) from the zero reaching time, a new output feedback control signal is suggested as

$$\begin{aligned}\dot{u}_i(t) &= -\left(T_i\bar{B}_i\right)^{-1}\Big\{\tilde{\eta}_i\left\|\sigma_i\right\|+\delta_{1i}\left\|\tilde{A}_{ii11}\right\|\left[\left\|\hat{\xi}_j\right\|+\varpi_j\right]\\&+\widehat{\varepsilon}_{1i}\left\|\sigma_i\right\|+\widehat{\varepsilon}_{2i}\left\|\dot{\sigma}_i\right\|+\left\|\tilde{X}_i\right\|\left\|P_i\right\|\left\|\dot{y}_i\right\|\\&+\sum_{\substack{j=1\\j\neq i}}^{L}\left[\widehat{\varepsilon}_{3i}\left(\left\|\hat{\xi}_i\right\|+\varpi_i\right)+\widehat{\varepsilon}_{4i}\left\|\sigma_i\right\|+\widehat{\varepsilon}_{5i}\left\|\dot{\sigma}_i\right\|\right]\\&+\left\|\dot{\vartheta}_i\right\|+\left\|\left[\tilde{X}_i\upsilon_i-\upsilon_i^2\right]\right\|\left\|P_i\right\|\left\|y_i(0)\right\|\exp(-\upsilon_it)\Big\}\\&\times\operatorname{sign}(\sigma_i(t)),\end{aligned}\tag{32}$$

where $\delta_{1i} = \left\|T_i\bar{A}_iS_i\bar{B}_i^{\perp}\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\right\|, \delta_{2i} = \left\|T_j\bar{H}_{ji}S_i\bar{B}_i^{\perp}\times\left(\bar{B}_i^{\perp T}S_i\bar{B}_i^{\perp}\right)^{-1}\right\|$, $\tilde{\eta}_i$ is positive constant, $\left\|\dot{\vartheta}_i(t)\right\|$ is external disturbance defined as (31) and $\widehat{\varepsilon}_{1i}, \widehat{\varepsilon}_{2i}, \widehat{\varepsilon}_{3i}, \widehat{\varepsilon}_{4i}, \widehat{\varepsilon}_{5i}$ are control gains that will determine later.

Theorem 3. Regard the uncertain interconnected plants with exogenous perturbations (1), suggest that the assumptions 1-4 are gratified. Then, the state variables of the closed-loop plants will hit the switching manifold surface $\sigma_i(y_i(t),t) = 0$ from the moment process under the control law (32) when scalar gains satisfy the following settings

$$\begin{aligned}&\widehat{\varepsilon}_{1i}\geq\delta_{1i}\left[\left\|\tilde{A}_{ii12}\right\|+\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\|\left\|\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|\right],\\&\widehat{\varepsilon}_{2i}\geq\left\|T_i\bar{A}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|,\\&\widehat{\varepsilon}_{3i}\geq\delta_{1i}\left\|\tilde{H}_{ji11}\right\|+\delta_{2i}\left(\left\|\tilde{A}_{ii11}\right\|+\left\|\tilde{H}_{ji11}\right\|\right),\\&\widehat{\varepsilon}_{4i}\geq\delta_{1i}\left(\left\|\tilde{H}_{ji12}\right\|+\left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\|\left\|\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right),\\&\quad+\delta_{2i}\left(\left\|\tilde{A}_{ii12}\right\|+\left\|\bar{B}_i^{\perp T}\bar{D}_i\right\|\left\|\bar{E}_i\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|\right)\\&\quad+\delta_{2i}\left(\left\|\tilde{H}_{ji12}\right\|+\left\|\bar{B}_j^{\perp T}\bar{F}_{ji}\right\|\left\|\bar{G}_{ji}\bar{B}_j\left(T_j\bar{B}_j\right)^{-1}\right\|\right)\\&\widehat{\varepsilon}_{5i}\geq\left\|T_j\bar{H}_{ji}\bar{B}_i\left(T_i\bar{B}_i\right)^{-1}\right\|.\end{aligned}\tag{33}$$

Proof of Theorem 3. Cogitate the candidate Lyapunov functional as

$$V_i=\sum_{i=1}^{L}\left\|\sigma_i(y_i(t),t)\right\|,\tag{34}$$

where direct differentiation of V_i results

$$\begin{aligned}
\dot{V}_i(\sigma_i) &= \sum_{i=1}^{L} \frac{\sigma_i(t)}{\|\sigma_i(t)\|} \Big\{ T_i\bar{A}_i S_i \bar{B}_i^{\perp} \big(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\big)^{-1} \\
&\times \dot{\xi}_i(t) + T_i \bar{A}_i \bar{B}_i \big(T_i\bar{B}_i\big)^{-1} \dot{\sigma}_i(t) \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{L} \Big[\sum_{i=1}^{L} \big[T_i \bar{H}_{ij} S_j \bar{B}_j^{\perp} \big(\bar{B}_j^{\perp T} S_j \bar{B}_j^{\perp}\big)^{-1} \dot{\xi}_j(t) \\
&+ T_i \bar{H}_{ij} \bar{B}_j \big(T_j \bar{B}_j\big)^{-1} \dot{\sigma}_j(t) \Big] \\
&+ \dot{\vartheta}_i(t) + \big(T_i\bar{B}_i\big)\dot{u}_i(t) + \tilde{X}_i \dot{\bar{s}}_i(y_i,t) \\
&+ \Big[\tilde{X}_i \upsilon_i - \upsilon_i^2\Big] \bar{s}_i(y_i,0)\exp(-\upsilon_i t) \Big\} \\
&\leq \sum_{i=1}^{L} \Big\{ \Big\| T_i\bar{A}_i S_i \bar{B}_i^{\perp} \big(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\big)^{-1} \Big\| \Big\| \dot{\xi}_i(t) \Big\| \\
&+ \Big\| T_i \bar{A}_i \bar{B}_i \big(T_i\bar{B}_i\big)^{-1} \Big\| \|\dot{\sigma}_i(t)\| \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{L} \Big[\Big\| T_j \bar{H}_{ji} S_i \bar{B}_i^{\perp} \big(\bar{B}_i^{\perp T} S_i \bar{B}_i^{\perp}\big)^{-1} \Big\| \\
&\times \Big\| \dot{\xi}_i(t) \Big\| + \Big\| T_j \bar{H}_{ji} \bar{B}_i \big(T_i \bar{B}_i\big)^{-1} \Big\| \|\dot{\sigma}_i(t)\| \Big] \\
&+ \Big\| \dot{\vartheta}_i(t) \Big\| + \frac{\sigma^T{}_i}{\|\sigma_i\|} \big(T_i\bar{B}_i\big)\dot{u}_i(t) \\
&+ \Big\| \tilde{X}_i \Big\| \|\dot{\bar{s}}_i(y_i,t)\| + \Big\| \Big[\tilde{X}_i \upsilon_i - \upsilon_i^2\Big] \Big\| \\
&\times \|\bar{s}_i(y_i,0)\| \exp(-\upsilon_i t) \Big\}.
\end{aligned} \tag{35}$$

Since

$$\begin{aligned}
\Big\|\dot{\xi}_i\Big\| &\leq \Big\|\tilde{A}_{ii11}\Big\| \Big[\Big\|\hat{\xi}_j(t)\Big\| + \varpi_j(t)\Big] + \Big[\Big\|\tilde{A}_{ii12}\Big\| \\
&+ \big\|\bar{B}_i^{\perp T}\bar{D}_i\big\| \Big\|\bar{E}_i\bar{B}_i\big(T_i\bar{B}_i\big)^{-1}\Big\|\Big] \|\sigma_i(t)\| \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{L} \Big\{ \Big\|\tilde{H}_{ji11}\Big\| \Big[\Big\|\hat{\xi}_i(t)\Big\| + \varpi_i(t)\Big] \\
&+ \Big[\Big\|\tilde{H}_{ji12}\Big\| + \big\|\bar{B}_j^{\perp T}\bar{F}_{ji}\big\| \\
&\times \Big\|\bar{G}_{ji}\bar{B}_j\big(T_j\bar{B}_j\big)^{-1}\Big\|\Big] \|\sigma_i(t)\| \Big\},
\end{aligned} \tag{36}$$

we have

$$\begin{aligned}
\dot{V}_i(\sigma_i) &\leq \sum_{i=1}^{L} \Big\{ \tilde{\eta}_i \|\sigma_i(t)\| + \delta_{1i} \Big\|\tilde{A}_{ii11}\Big\| \Big[\Big\|\hat{\xi}_j(t)\Big\| + \varpi_j(t)\Big] \\
&+ \widehat{\varepsilon}_{1i}\|\sigma_i\| + \widehat{\varepsilon}_{2i}\|\dot{\sigma}_i\| + \sum_{\substack{j=1\\ j\neq i}}^{L} \Big[\widehat{\varepsilon}_{3i}\Big(\Big\|\hat{\xi}_i(t)\Big\| + \varpi_i(t)\Big) \\
&+ \widehat{\varepsilon}_{4i}\|\sigma_i\| + \widehat{\varepsilon}_{5i}\|\dot{\sigma}_i\|\Big] + \Big\|\dot{\vartheta}_i\Big\| + \frac{\sigma^T{}_i(t)}{\|\sigma_i(t)\|}\big(T_i\bar{B}_i\big)\dot{u}_i(t) \\
&+ \Big\|\tilde{X}_i\Big\| \|\dot{\bar{s}}_i\| + \Big\|\Big[\tilde{X}_i\upsilon_i - \upsilon_i^2\Big]\Big\| \|\bar{s}_i(y_i,0)\| \exp(-\upsilon_i t)\Big\},
\end{aligned} \tag{37}$$

where the control gains $\widehat{\varepsilon}_{1i}, \widehat{\varepsilon}_{2i}, \widehat{\varepsilon}_{3i}, \widehat{\varepsilon}_{4i}$ and $\widehat{\varepsilon}_{5i}$ are detailed in (33). Now, by replacing the control signal (32) into (37), we can see that $\dot{V}_i(\sigma_i) = \sum_{i=1}^{L} \tilde{\eta}_i \|\sigma_i(t)\|$, $\tilde{\eta}_i$ is positive constant. Thus, the state variables of the system arrive the sliding manifold from the zero-reaching time for all $t \geq 0$. Proof of Theorem 3 is finished.

4. Numerical simulation

In this part, the above proved attainments are verified by the numerical example that improved from the research [22]. The mathematical simulation of the mismatched uncertain interconnected plants (1) is characterized as.

Subsystem I: $i = 1, j = 2, n_1 = 3, m_1 = 2$, and the dynamics is given as

$$\begin{aligned}
\dot{x}_1(t) &= \big[\bar{A}_1 + \bar{D}_1\Xi_1(x_1(t),t)\bar{E}_1\big]x_1(t) \\
&\quad + \bar{B}_1\left[u_1(t) + \psi_1(x_1(t),t)\right] \\
&\quad + \big[\bar{H}_{12} + \bar{F}_{12}\Xi_{12}(x_2(t),t)\bar{G}_{12}\big]x_2(t), \\
y_1(t) &= \bar{C}_1 x_1(t),
\end{aligned} \tag{38}$$

where $\bar{A}_1 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 1 & -0.75 \end{bmatrix}$, $\bar{B}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\bar{H}_{12} = \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}$, and $\bar{C}_1 = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. To display the usefulness of the suggested decentralized CSORSMC, it is assumed that the exogenous perturbation is $\Big\|\vartheta_1(t)\Big\| \leq 0.01\|y_1\| + 0.21\|y_1\| \times (\|x_1\|) + 0.04\|y_1\|(\|x_1\|)^2$ the mismatched uncertainty in state matrix and the interconnection respectively are $\bar{D}_1\Xi_1(x_1(t),t)\bar{E}_1$, where

$\bar{D}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \Xi_1(x_1(t), t) = 0.1\sin(0.1t), \bar{E}_1 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, and $\bar{F}_{12}\Xi_{12}(x_2(t), t)\bar{G}_{12}$, where $\bar{F}_{12} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \Xi_{12}(x_2(t), t) = 0.3\sin(0.1t), \bar{G}_{12} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$.

Subsystem II: $i = 2, j = 1, n_2 = 3, m_2 = 2$ and the dynamics is given as

$$\begin{aligned} \dot{x}_2(t) &= \left[\bar{A}_2 + \bar{D}_2\Xi_2(x_2(t), t)\bar{E}_2\right] x_2(t) \\ &\quad + \bar{B}_2\left[u_2(t) + \psi_2(x_2(t), t)\right] \\ &\quad + \left[\bar{H}_{21} + \bar{F}_{21}\Xi_{21}(x_1(t), t)\bar{G}_{21}\right] x_1(t), \\ y_2(t) &= \bar{C}_2 x_2(t), \end{aligned} \tag{39}$$

where $\bar{A}_2 = \begin{bmatrix} -0.1 & 1 & 0.2 \\ 1 & 1 & -1 \\ 0.5 & 1 & 0.1 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix}, \bar{H}_{21} = \begin{bmatrix} -0.2 & 0 & -0.1 \\ 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0 \end{bmatrix}$, and $\bar{C}_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. To display the usefulness of the suggested decentralized CSORSMC, it is assumed that the exogenous perturbation is $\left\|\dot{\vartheta}_2(t)\right\| \leq 0.01\|y_2\| + 0.21\|y_2\| \times (\|x_2\|) + 0.04\|y_2\|(\|x_2\|)^2$ the mismatched uncertainty in state matrix and the interconnection respectively are $\bar{D}_2\Xi_2\bar{E}_2$, where $\bar{D}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \Xi_2 = 0.1\sin(0.1t), \bar{E}_2 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, and $\bar{F}_{21}\Xi_{21}(x_1(t), t)\bar{G}_{21}$, where $\bar{F}_{21} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \Xi_{21} = 0.4\sin(0.3t), \bar{G}_{21} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$. The initial situations of two subsystems are chosen as $x_1(0) = \begin{bmatrix} 0.1 & -0.1 & 0.01 \end{bmatrix}^T$ and $x_2(0) = \begin{bmatrix} 0.1 & -0.1 & 0.2 \end{bmatrix}^T$ respectively. By using the MATLAB software, the simulation consequences are respectively portrayed from Figs. 2-7 including the state variables of two subsystems, the ROSME, error dynamics, the upper limit of the error, the switching manifold and the novel CSORSMCs. From the above-mentioned simulation of the attained achievements, we can realize that the suggested approach is effective in dealing with the chattering destruction and the reaching phase removal problems for a class of uncertain interconnected systems with extended perturbations and mismatched uncertainties.

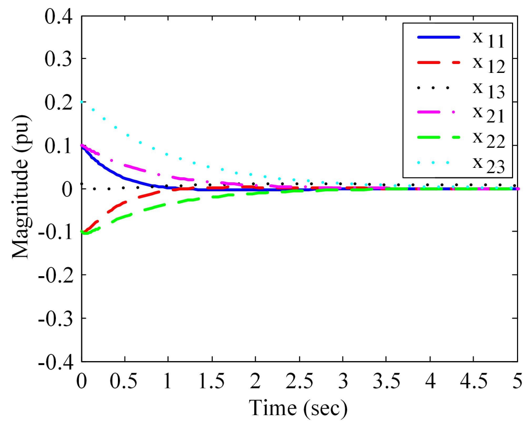

Fig. 2: Time answer of the plant states of two subsystems.

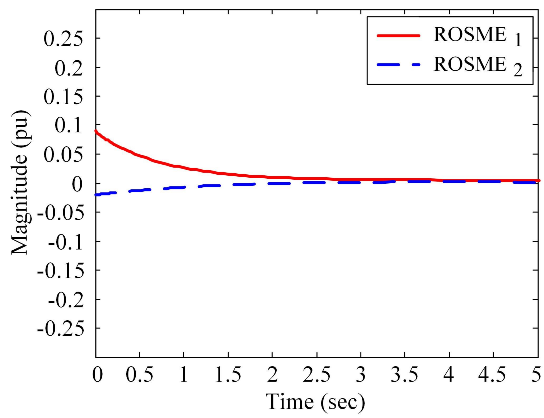

Fig. 3: Time reactions of the observer of two subsystems.

5. Conclusions

In this research, a decentralized robust stabilization and the chattering avoidance problem of the complex interconnected plants with mismatched parameter uncertainties in interconnections and state matrix have been explored in which the external perturbation is extended. Especially,

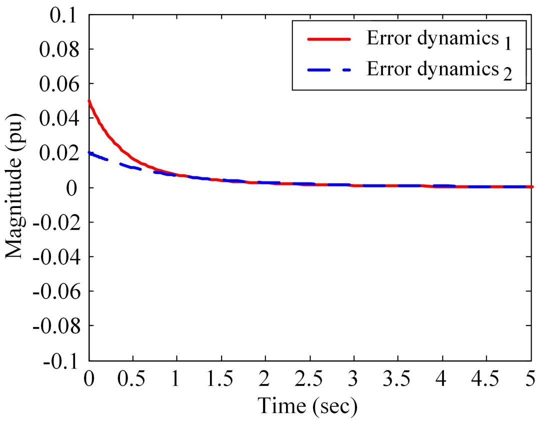

Fig. 4: Time history of the observer errors.

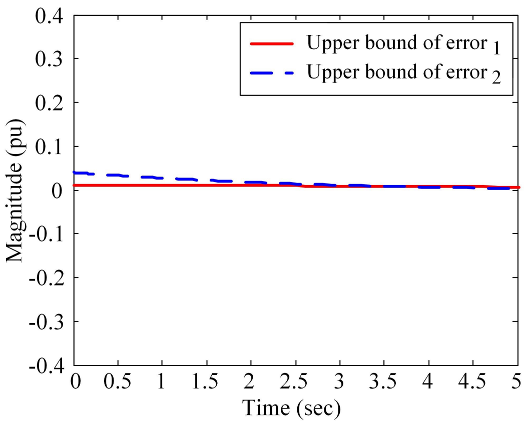

Fig. 5: The trajectories of the error's upper limit.

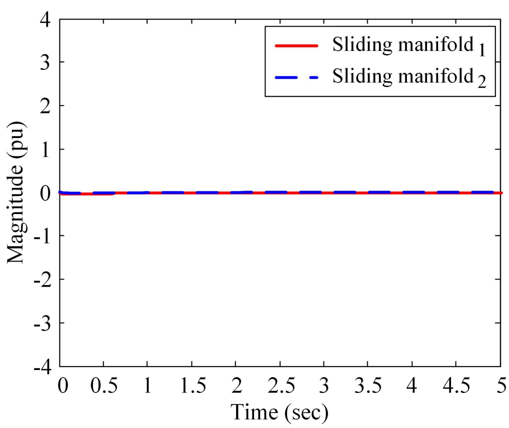

Fig. 6: Time history of the switching manifolds.

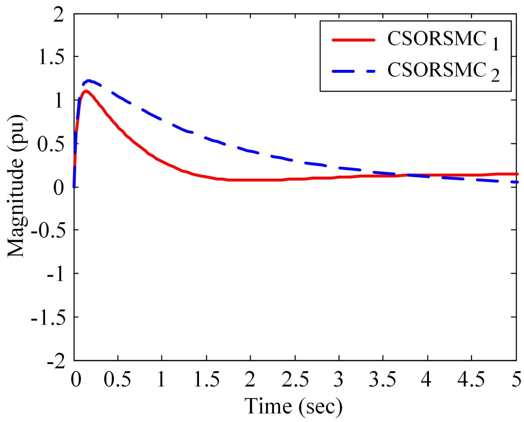

Fig. 7: Time responses of the suggested controllers.

this research has presented the improved SMC without reaching phase such that the plant's robustness is guaranteed when its whole state variables always start from the switching manifold surface. The ROSME has been established to guess the immeasurable states. By employing the ROSME tool and the Moore-Penrose inverse technique, a new decentralized CSORSMC not only resolves the single-phase complex interconnected problem but also cancels the undesired high frequency fluctuation in control signal. Moreover, the reduced-order interconnected plants in sliding mode are asymptotically stable by means of the Lyapunov stability theory and the well-known LMI technique. In addition, instance simulation is specified to confirm the practicability and usefulness of the key achievements. However, our study has not considered the time-delay effect which leads to an unsteadiness and/or diminishes the plant performance. Hence, the development of the offered method to other more general plants involving unidentified time-varying delays could be the future trend.

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