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UNCERTAINTY QUANTIFICATION OF BUCKLING PROPERTIES FOR IMPERFECT SPHERICAL SHELL SUBJECTED TO EXTERNAL PRESSURE

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Abstract. *This study conducts a sensitivity analysis of the buckling properties of imperfect spherical shells, accounting for the uncertainty parameters of a spherical shell under external pressure. These parameters include geometric imperfections, thickness variations, material property variations, and boundary condition variations. A numerical analysis method utilizing ABAQUS is employed, incorporating these various uncertainty imperfection parameters into the numerical models The study examines the influence of these factors on critical buckling load, mode shapes, and stress distribution. The results demonstrate that an increase in shell thickness leads to an increase in the spherical shell buckling load. Correlations between the imperfection parameters and the critical buckling load are established, providing crucial insights for design and analysis purposes. Furthermore, the study reveals the occurrence of various buckling modes and failure mechanisms, which is crucial for understanding the structural behavior of imperfect spherical shells subjected to external pressure.*

Keywords: *Spherical shell, Buckling, Imperfections, Uncertainty quantification, External pressure.*

1. Introduction

Spherical shells are widely used in many engineering applications, such as pressure vessels, storage tanks, domes, and submarine hulls [1]. Perfect shells are considered ideal structures because they typically have a greater load carrying capacity compared to shells that have deviations in geometry, material behavior, and boundary conditions. The spherical shell is created in accordance with the most recent building standards, including (i) the British Standard, (ii) the American Bureau of Shipping (ABS), (American Bureau of Shipping, 2021) and (iii) the European Convention for Constructional Steelwork. These design codes, however, provide significant variations in their recommendations for the minimum thickness of the spherical shell, necessitating further research to develop a consistent method for estimating the ultimate strength of spherical pressure hulls under external pressure [1].

Initial imperfections, such as geometrical deviations, material property variations, and boundary condition variations, can significantly impact the buckling behavior and load-carrying capacity of spherical shells [2]. Numerous studies have been conducted to investigate the effects of these imperfections on the buckling behavior of spherical shells [3, 4]. Imperfect shell structures often experience various loading conditions, particularly in engineering applications, with external pressure being the most critical variable. The ultimate strength and stability of an externally pressured spherical shell are significantly influenced by its shape, material properties, pre-buckling deformations, and geometric defects. Numerous experimental and numerical investigations have been carried out to study the elastic-plastic buckling of spherical shells under external pressure, focusing on factors such as wall thickness, yield strength, and imperfection size. These studies have provided valuable insights into the buckling behavior of spherical shells, but they often rely on simplified assumptions or idealized geometries.

The present study aims to address this gap by conducting a comprehensive uncertainty quantification analysis that considers the combined effects of various imperfection parameters on the buckling properties of spherical shells subjected to external pressure. The influence of geometric imperfections, thickness variations, material property variations, and boundary condition variations on critical buckling load, mode shapes, and stress distribution is investigated using finite element analysis. Specifically, the study aims to establish correlations between the imperfection parameters and the critical buckling load, providing crucial insights for the design and analysis of imperfect spherical shells under external pressure. To achieve this objective, detailed numerical models are developed using the ABAQUS finite element software, which allows for the incorporation of various uncertainty parameters, such as geometric imperfections, thickness variations, material property variations, and boundary condition variations. The results of the study will contribute to a better understanding of the structural behavior of imperfect spherical shells and the development of improved design methodologies for such structures. The paper is structured as follows: Section 1. provides a literature review on the existing research on the buckling of imperfect spherical shells. Section 2. outlines the numerical methodology used in this study, including the finite element modeling and the incorporation of uncertainty parameters. Section 3. presents the results and discussions, highlighting the key findings. Section 5. concludes the paper and provides recommendations for future research.

2. Literature review

Perfect shells may be optimal structures in the sense that their load carrying capacity is usually larger than that of shells which show deviations in geometry, material behavior and boundary. While in imperfect shell structures, the shell is probably subjected to a variety of loading circumstances, especially in engineering applications, with external pressure turning out to be the most significant. The buckling behavior of spherical shells under external pressure has been a topic of extensive research for decades. Many studies have been conducted to investigate the effects of various parameters, such as geometric imperfections, thickness variations, material properties, and boundary conditions, on the critical buckling load and deformation behavior of spherical shells [2, 5–7].

The work by Abbasi et al. [8] focused on probing the buckling of pressurized spherical shells, by analyzing the nonlinear force-indentation response of imperfect shells at different pressurization levels. They combined experiments, finite element modeling, and existing results from classic shell theory to characterize the nonlinear force-indentation response of imperfect shells at different pressurization levels. The study by Tall et al. [7] investigated the elastoplastic buckling and collapse of spherical shells under combined external pressure and circumferential shear loadings, providing a comprehensive understanding of the imperfection sensitivity of such structures. The work by Koga and Hoff [5] focused on the axisymmetric buckling of initially imperfect complete spherical shells. They investigated the buckling and post-buckling behavior of complete spherical shells under the assumption that both the unintentional, random initial deviations from the exact shape and the following elastic deformations are symmetric to some radius of the shell. The study by Darmawan and Tedianto [6] analyzed the critical buckling load of spherical shells under various geometric imperfections, including the use of the perturbation cutout method to represent the most realistic imperfections on shells. By incorporating uncertainty quantification into the sensitivity analysis, the study systematically evaluates the impact of these uncertainties on the buckling behavior of spherical shells. This approach not only enhances the robustness and reliability of the numerical models but also provides valuable insights into design and analysis, ensuring that the models can accurately predict real world behavior under different conditions.

2.1. Geometrical imperfections

According to research by Fazlalipour et al. [9], geometric imperfections significantly impact the buckling strength of structures. Among the various types of imperfections, geometric flaws have the most pronounced effect on reducing buckling strength. The buckling properties of a spherical shell depend on geometric parameters such as the radius and aspect ratio. Fazlalipour et al. [9] found that shells with larger radii or lower aspect ratios are more resistant to failure under external pressure. The imperfection shape and magnitude also strongly influence the buckling behavior. As per Donnell theory, the initial geometric imperfections can be described by a trigonometric series or a superposition of multiple sinusoidal waves. To account for initial imperfections, a knockdown factor is introduced. This factor represents the ratio between the global buckling loads of a thin-shell structure without and with imperfections. It is particularly sensitive for shells with specific height-to-base radius designs. Hemispherical shells experience the most severe drop in buckling strength due to initial flaws. If the applied load exceeds the structure's critical buckling load, catastrophic failure can occur. Unstable post-buckling behavior may lead to a reduction in the critical buckling load. Geometric defects can substantially lower the critical load. Imperfections can introduce stress concentrations and localized deformations, which can impact the load-carrying capacity of the shell. Consequently, imperfect shells tend to buckle at lower applied loads compared to perfect shells. Understanding and effectively managing geometric imperfections are essential for ensuring the stability and safety of shell structures.

2.2. Material properties

The material properties of a spherical shell significantly impact its buckling behavior, as they affect the material capacity to resist deformation and failure. Consequently, the critical buckling load, failure mode, and post-buckling behavior of the shell are also influenced. The buckling behavior is influenced by the shell's material characteristics, such as its elastic modulus and yield strength. Stronger materials provide higher buckling resistance when subjected to external pressure. Buckling is a phenomenon that occurs when the applied load exceeds the critical load, and the critical buckling load typically increases with the Young's modulus. Various materials have been employed in the fabrication of spherical shells, with the material characteristics tailored to the intended application. For instance, titanium alloys are commonly used for pressure hulls in manned submersibles due to their high strength-to-weight ratio and corrosion resistance. In summary, the quantification of uncertainty in the buckling properties of imperfect spherical shells under external pressure is crucial for the design and safety assessment of such structures.

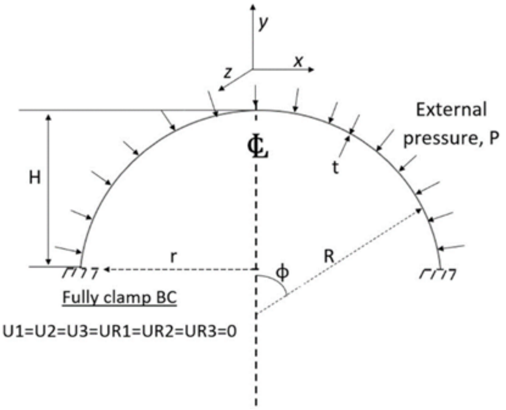
2.3. Thickness variation

Beyond the influence of initial geometric imperfections and material properties, the thickness of the spherical shell is another critical parameter that affects its buckling behavior under external pressure. Variations in the thickness of a spherical shell can result in geometric irregularities that have the potential to impact the critical buckling load, buckling mode, and post-buckling behavior. Thinner shells tend to be more prone to buckling failure, as they offer less resistance to deformation. Conversely, thicker shells generally exhibit higher critical buckling loads but may also be subject to other failure modes, such as plastic collapse or material yielding. The presence of a non-uniform-thickness shell can have a substantial impact on the critical buckling load of the shell. According to the study by Ait L'Hadj et al. [10], non-uniform thickness distribution can cause variations in stiffness across a structure, resulting in localized stress concentrations.

Compared to a uniform-thickness shell, the presence of stress concentrations in a non-uniform thickness shell has the potential to decrease its critical buckling load. Shells with reduced thickness are more susceptible to buckling due to their decreased ability to resist deformation. Furthermore, non-uniform thickness introduces fluctuations in the shell's stiffness and strength. The non-uniform thickness of a spherical shell can result in stress concentrations, altered buckling behavior, non-uniform deformation patterns, and reduced buckling strength. Consequently, the design of spherical shells requires a careful balance between the shell thickness and other parameters to optimize the structure's load-carrying capacity and safety.

2.4. Boundary conditions

The boundary conditions applied to a spherical shell can significantly impact its buckling behavior under external pressure. Shells with different boundary conditions, such as clamped, simply supported, or free edges, may exhibit distinct buckling modes and critical loads. Spherical shells supported at the edges, such as those found in pressure vessels and submarine hulls, typically exhibit higher critical buckling loads compared to shells with free edges. The boundary conditions can also influence the post-buckling response of the shell, affecting its load-carrying capacity beyond the critical load. When the shell is simply supported at the edges, the boundary conditions restrict the radial displacements and rotation, leading to a higher critical buckling load. In contrast, a shell with free edges experiences lower critical buckling loads due to the lack of such constraints. The specific boundary conditions imposed on a spherical shell are often dictated by the intended application and the surrounding structural elements. Understanding the impact of boundary conditions on the buckling behavior of imperfect spherical shells is crucial for the accurate prediction of their load-carrying capacity and structural integrity.



Type of BC	U1	U2	U3	UR1	UR2	UR3
1	c	c	c	c	c	c
2	c	c	c	f	c	c
3	c	c	f	c	c	c
4	c	c	f	f	c	c

Fig. 1: Schematic diagram of load and type of boundary condition of the externally pressurized spherical shell in finite element analysis (FEA). Note c denotes that the variable is set to zero and note f, variable is set to free (Ismail et al., [11]).

2.5. Modeling techniques and uncertainty quantification

Finite element analysis is a widely used numerical technique for the analysis of shell structures, as it can account for the nonlinear behavior and complex load-deformation characteristics of these systems. Finite element models can be employed to study the influence of geometric imperfections, material properties, and boundary conditions on the critical buckling load and post-buckling behavior of spherical shells. Beyond the deterministic analysis of spherical shell buckling, the quantification of the associated uncertainties is crucial for the reliable design and assessment of such structures. According to Ait L'Hadj et al. [10], the uncertainties in the geometric imperfections, material properties, and thickness variations can significantly impact the predicted critical buckling load of spherical shells (Tall et al., [7]) (Ma et al., [12]). Probabilistic approaches, such as Monte Carlo simulations, can be utilized to assess the influence of these uncertainties on the buckling behavior of spherical shells. By considering the statistical distributions of the relevant parameters, these techniques can provide a more comprehensive understanding of the shell load-carrying capacity and the associated reliability.

3. Methodology

The present study employs finite element analysis (FEA) using the ABAQUS software to investigate the uncertainty quantification of buckling properties for imperfect spherical shells subjected to external pressure. Sensitivity analysis: A method that the finite element models are developed to incorporate various uncertainty parameters, including geometric imperfections, thickness variations, material property variations, and boundary condition variations. The geometric model of the spherical shell is first created, with the radius,R and thickness, t of the shell defined based on the design parameters. This allows for the evaluation of the sensitivity of the critical buckling load and deformation behavior to the different uncertainty parameters. Next, the material properties, including the elastic modulus, Poisson's ratio, and yield strength, are defined, and the appropriate boundary conditions are applied to the model. Medium carbon steel is adopted with a Young's Modulus, E of 207 GPa and a poisson's ratio, v of 0.28 for all imperfect spherical shell models except for material properties variation models. The shell is modeled using four-node quadrilateral shell elements (S4R) with reduced integration.

Linear buckling analysis (LBA) method is applied via ABAQUS software to investigate the buckling behavior of imperfect spherical shells subjected to external pressure. The analysis considers the nonlinear behavior of the shell, including the effects of large deformations and material plasticity. The numerical model is based on an elastic-plastic material model performed by Błachut [13]. In this study, we consider a thin imperfect spherical shell with radius-to-thickness ratio, R/t ranges from 100 to 1000. Lanczos algorithms are employed to compute critical eigenvalues related to buckling or dynamic behavior and used to approximate the eigenvalues and eigenvectors of a large sparse matrix. The loading condition is set at -1 MPa to simulate external pressure on all imperfect spherical shell models. The boundary conditions are determined based on Ismail et al. [11], which includes fully clamped, simply supported, fully free, or mixed ends for the numerical analysis as listed in Fig. 1 .

The behaviour of spherical shells under load is described using the equations of elasticity and thin shell theory. The exact expression for the critical buckling pressure $P_o$of these thin spherical shells is Eq. 1:

$$P_0 = \frac{2E}{\sqrt{3(1-v)}}\left(\frac{t}{R}\right)^2 \tag{1}$$

where E refers to the Young's modulus, v defines the poisson's ratio whilst t and R refer to the thickness and radius of spherical shell. The plastic buckling, in the case of yield stress approximate value for a plastic buckling pressure, P_Y, is expressed as in Eq. 2:

$$P_Y = 2\sigma_{Yield}\left(\frac{t}{R}\right) \tag{2}$$

The estimated collapse pressure, P_{coll} using the design code of PD 5500 is given by as in Eq. 3:

$$\left(\frac{1}{P_{PD5500}}\right)^2 = \frac{1}{0.3P_{cr}} + \left(\frac{1}{P_{Yield}}\right)^2 \tag{3}$$

4. Results and discussions

The study utilizes how sensitivity analysis changes in a model parameter affect its output. This study also extends the work by Ismail et al. [11] by utilizing the uncertainty parameters that affect the buckling behavior of imperfect spherical shell model. The imperfection analysis focuses on the uncertainty parameter of a spherical shell under external pressure, which includes (i) geometric imperfection and thickness variation, (ii) material property variation, and (iii) boundary condition variation. The influence of the buckling load on the uncertainty parameters of a spherical shell under external pressure was evaluated numerically using Abaqus using the linear buckling analysis (LBA) method, with the specified input parameter range.

This study employs the exact critical buckling pressure to validate the convergence of the perfect shell element. Specifically, a thin spherical shell with a radius-to-thickness ratio of 100mm is considered. By implementing various mesh designs, the critical buckling pressure is computed to assess the convergence characteristics. Table 1 summarizes the normalized buckling pressures, i.e., the ratio of the critical buckling pressure to the exact solution (P_{cr}/P_o). The results indicate that a mesh design with 6084 elements number is required to achieve converged solutions. This mesh size will be utilized for generating all the subsequent buckling results presented in this work.

Tab. 1: Convergence study of numerical results with respective number of elements used for a spherical thin shell.

Mesh size	No. of element	Normalized buckling (P_{cr})
1	24964	24.790 MPa
2	6084	24.875 MPa
5	1024	24.999 Mpa
8	400	25.293 MPa
10	256	25.692 MPa

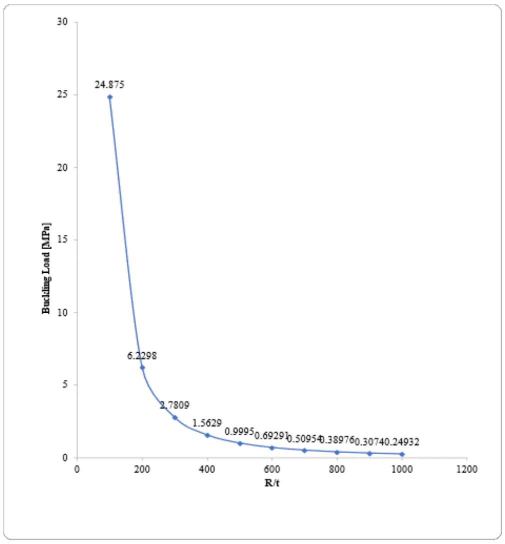


Fig. 2: Curve fitting of normalized critical and plastic buckling pressures of sphericall shells with various radius-to-thickness ratios.

4.1. Geometric imperfection and thickness variation

The data in Table 2 compares normalized buckling pressures, plastic buckling pressures, and collapse pressures of spherical shells for various radius-to-thickness (R/t) ratios using results from both linear buckling analysis (LBA) method and formula-derived values. Across all categories, the values demonstrate a strong correlation between finite element analysis (FEA) results and analytical formulae, with minimal percentage differences, often less than 0.1%. The table shows that as the R/t ratio increases, all three pressures—buckling, plastic buckling, and collapse—gradually decrease. The normalized buckling pressure P_{cr}/P_o represents the critical pressure at which a spherical shell undergoes elastic instability. Theoretical models predict that as the R/t ratio increases, the shell becomes more prone to buckling under lower pressures due to reduced stiffness. This is reflected in the data, where normalized buckling pressure decreases significantly, from 24.875 MPa at $R/t = 100$ to 0.24932 MPa at $R/t = 1000$. The near-perfect agreement between finite element analysis (FEA) and formula-derived values suggests that the chosen equation accurately models the relationship between critical buckling pressure and R/t ratio.

Similarly, the normalized plastic buckling pressure $P_{Y'}/P_Y$ refers to the pressure at which the shell yields plastically before buckling. As

Tab. 2: Normalized buckling pressures, plastic buckling pressure, and collapse pressure of spherical shells with various radius-to-thickness ratios.

Radius-to-thickness ratio, R/t (mm)	Normalized buckling pressure, P_{cr}/P_e (MPa)		Percentage difference $\frac{\lvert A-B\rvert}{\frac{A+B}{2}} \times 100$	Normalized plastic buckling pressure, P'_{cr}/P_e (MPa)		Percentage difference $\frac{\lvert C-D\rvert}{\frac{C+D}{2}} \times 100$	Normalized Collapse Pressure, P_{coll}/P_{call} (MPa)		Percentage difference $\frac{\lvert E-F\rvert}{\frac{E+F}{2}} \times 100$
	FEA (A)	Formula given by Eq. 1 (B)		FEA (C)	Formula given by Eq. 2 (D)		FEA (E)	Formula given by Eq. 3 (F)	
100	24.875	24.880	0.020%	6.070	6.070	0%	0.161	0.161	0%
200	6.2298	6.230	0.003%	3.035	3.035	0%	0.644	0.644	0%
300	2.7809	2.781	0.004%	2.027	2.027	0%	1.443	1.443	0%
400	1.5629	1.563	0.006%	1.518	1.518	0%	2.576	2.576	0%
500	0.9995	1.0	0.050%	1.214	1.214	0%	4.025	4.025	0%
600	0.69291	0.693	0.013%	1.011	1.011	0%	5.801	5.801	0%
700	0.50954	0.510	0.090%	0.759	0.759	0%	10.305	10.305	0%

with elastic buckling, the plastic buckling pressure decreases with higher R/t ratios, reflecting the weakening structural capacity of thinner shells. For all R/t ratios in the table, finite element analysis (FEA) and theoretical formula values are identical, indicating that the plastic buckling equation is robust and reliably captures the onset of plastic deformation in spherical shells. Collapse pressure $P_{coll'}$ / P_{coll} represents the final pressure at which the structure fails completely. The collapse pressure also shows a decline with increasing R/t ratios. The theoretical model used to calculate collapse pressure is in perfect alignment with finite element analysis (FEA) results, as the percentage difference remains 0% across all ratios. This highlights that the formula accurately represents the collapse behavior of spherical shells, particularly under high R/t ratios, where shells are more susceptible to complete structural failure under lower loads.

The decreasing trend in all pressures is consistent with classical shell theory, which states that thinner shells (higher R/t ratios) have lower buckling and collapse strengths due to their reduced resistance to compressive forces. The close match between theoretical and finite element analysis (FEA) results underscores the validity of the applied equations in practical applications, ensuring accurate design predictions for spherical structures under critical loading conditions. It was also observed that an increase in the shell thickness can lead to a higher critical buckling pressure, as the shell becomes more resistant to deformation as demonstrated by Darmawan and Tedianto [6]. Fig. 2 depicts the correlation between the critical buckling load, $P_{cr}$with the variation of R/t ratios. Based on the results in Table 1 and Fig. 2, it is observed that spherical shells with thicker walls exhibit a greater capacity to withstand external pressure before reaching the critical buckling load. As the shell thickness increases, the structure becomes more resistant to deformation and can sustain higher levels of external pressure without experiencing buckling failure. This is because thicker-walled spherical shells have a higher moment of inertia and can better resist the compressive stresses induced by the external pressure as demonstrated by Ma et al. [12]. The critical buckling load for a spherical shell model with a shell thickness below 1 mm signifies that the structure is unstable. Larger imperfection depths lead to a significant reduction in the critical buckling pressure, as the shell becomes more vulnerable to instability. This result agrees with Tall et al. [7] which found that the critical buckling pressure of the spherical shell is highly sensitive to the presence and magnitude of geometric imperfections.

4.2. Material properties variation

Table 3 and Fig. 3 represent comparative results for normalized critical buckling pressure, P_{cr}/P_o plastic buckling pressure, $P_{Y'}/P_Y$ and collapse pressure, $P_{coll'}/\ P_{coll}$for spherical shells made from various steel materials, highlighting significant differences in their structural performance. The materials assessed include AISI 1040, AISI 1340, AISI 2340, AISI 3140, AISI 4140, AISI 4340, AISI 5140, and AISI 6140. AISI 4340 performs exceptionally well with a normalized buckling pressure of 43.622 MPa, highlighting its superior resistance to elastic instability. AISI 4140 follows closely at 36.292 MPa, demonstrating that it can handle significant buckling loads. On the lower end, AISI 1340 and AISI 1040 have normalized buckling pressures of 22.832 MPa and 24.056 MPa, respectively, making them more suitable for applications where the risk of buckling is lower. AISI 4340 stands out with the highest normalized plastic buckling pressure at 17.220 MPa, followed by AISI 4140 at 13.100 MPa. These steels are capable of withstanding significant compressive loads before plastic deformation begins, which is critical in heavy-duty structural applications. In contrast, AISI 1040 and AISI 1340 exhibit much lower normalized plastic buckling pressures at 8.400 MPa and 8.680 MPa, respectively, indicating that they will begin to deform plastically under relatively lower pressures. AISI 1040 has the highest normalized collapse pressure at 0.1527, indicating relatively better resistance to catastrophic failure. However, AISI 4340 and AISI 4140, while excelling in buckling resistance, show the lowest normalized collapse pressures at 0.0797 and 0.0976, respectively, suggesting that although these materials can withstand higher buckling pressures, they might be more prone to collapse once the critical load is reached.

To conclude, AISI 4340 and AISI 4140 stand out for their high buckling resistance and plastic buckling pressures, making them ideal for heavy-duty and high-stress applications, despite their relatively lower collapse pressures. In contrast, AISI 1040 and AISI 1340 offer lower performance in these areas but provide better collapse resistance, suggesting their use in more moderate load-bearing applications where cost and overall failure resistance are prioritized over buckling resistance. These insights are essential for material selection in engineering, especially for structures requiring both high load capacity and resilience to buckling and collapse.

Tab. 3: Critical, plastic buckling pressures and collapse pressures of a thin spherical shell with various medium carbon steel properties (R/t = 100; n = 1, v = 0.28, BC = Type 1)

AISI / Type	**E (GPa)**	**Yield (MPa)**	**Pcr / Po (MPa)**	**PY' / PY (MPa)**	**Pcoll'/ Pcoll (MPa)**
1040/ Plain Carbon	200	420	24.056	8.400	0.1527
1340 / Mn Steel	190	434	22.832	8.680	0.1591
2340 / Ni Steel	200	620	24.034	12.400	0.1451
3140 / Cr-Ni Steel	212	485	25.476	9.700	0.1413
4140 / Cr-Mo steel	302	655	36.292	13.100	0.0976
4340 / Ni-Cr-Mo steel	363	861	43.622	17.220	0.0797
5140 / Cr steel	229	427	27.519	8.540	0.1347
6140 Cr-V steel	269	615	32.326	12.300	0.1096

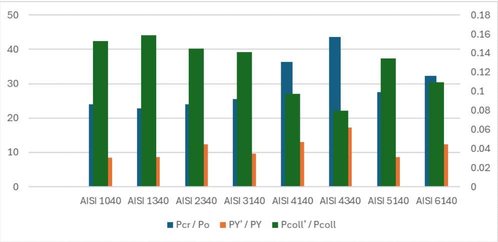


Fig. 3: Comparative results of normalized (a) critical buckling pressure; (b) plastic buckling pressure and (c) collapse pressure with various material properties (R/t =100; eigenmode, n= 1; BC type =1).

4.3. Boundary condition variation (BC)

The study also examined the influence of boundary conditions (BC) on the buckling behavior of the imperfect spherical shell as illustrated in Fig. 5. Table 2 presents the buckling analysis outcomes for eigenmode $n=1$, considering different boundary conditions (BC type 1 to BC type 4). The buckling load for Boundary condition type 1 is similar to that of boundary conditions type 3. The buckling load for boundary conditions type 2 and type 4 shows a minimal discrepancy of around 0.001 in terms of inaccuracy. This study agrees with Fazlalipour et al. [9] findings regarding the influence of buckling mode and critical buckling load on a spherical shell under external pressure. It denotes that the full clamped boundary conditions (BC type 1) increase imperfect spherical shell rigidity and minimize deformation. This boundary condition also limits shell translation and rotation at its edges which increases the stiffness of the shells and limits their ability to deform. An increase in thickness (t) leads to a corresponding increase in the critical buckling load, indicating a higher level of resistance to buckling, as inferred from the analysis of changes in boundary conditions. It was found that the choice of boundary conditions, such as the degree of edge restraint, can significantly affect the critical buckling pressure of the shell as demonstrated by Koga and Hoff [5].

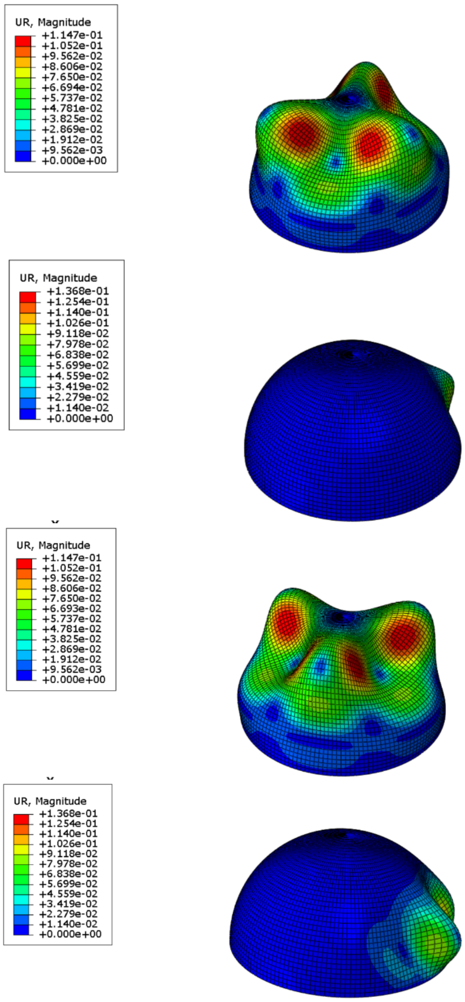
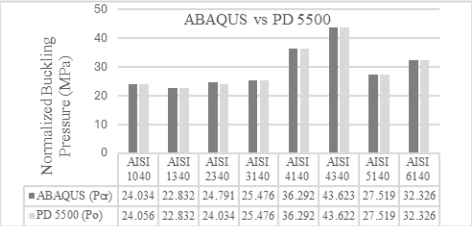


Fig. 4: Buckling mode shapes of spherical shells tested on plain carbon material with variation of boundary conditions for eigenmode ($n = 1$).

4.4. The validation of the numerical model

The spherical shell buckling load analysis compares the dataset obtained from numerical analysis using the FEA software Abaqus, specifically through linear buckling analysis (LBA), with the results calculated using design code of the British Standard (PD 5500, 2009) in (Eq. 1). The spherical shell models considered for this analysis maintain a fixed radius of 500 mm. Eight different materials were assessed for validation: AISI 1040, AISI 1340, AISI 2340, AISI 3140, AISI 4140, AISI 4340, AISI 5140, and AISI 6140. Through the analysis in Fig.??, it was found that PD 5500 and ABAQUS results exhibit a high degree of agreement, thereby validating the reliability and uniformity of both analytical methodologies. A correlation coefficient of 1 indicates a perfect positive correlation between the buckling load (Pcr) calculated using the British Standard (PD 5500, 2009) and the numerical analysis via Abaqus, suggesting a robust linear relationship. Consequently, both methods are validated for identifying the buckling load capacity of pressure vessels.



	AISI 1040	AISI 1340	AISI 2340	AISI 3140	AISI 4140	AISI 4340	AISI 5140	AISI 6140
■ ABAQUS (Pcr)	24.034	22.832	24.791	25.476	36.292	43.623	27.519	32.326
□ PD 5500 (Po)	24.056	22.832	24.034	25.476	36.292	43.622	27.519	32.326

Fig. 5: Comparison of the buckling load results between numerical and analytical approaches.

5. Conclusion

The study investigated the influence of geometric imperfections, thickness variations, material property variations, and boundary condition variations on the critical buckling load, deformation behavior, and stress distribution of imperfect spherical shells under external pressure. Here are some key results highlighted in the study:

- A strong correlation between finite element analysis (FEA) results and analytical formulae, with minimal percentage differences often less than 0.1%, suggesting the accuracy of the chosen equation in modeling the relationship between critical buckling pressure and the radius to thickness (R/t) ratio.
- The choice of boundary conditions, such as the degree of edge restraint, significantly affected the critical buckling pressure of the shell, indicating the importance of boundary conditions in the buckling behavior of imperfect spherical shells.
- An increase in thickness (t) led to a corresponding increase in the critical buckling load, indicating a higher level of resistance to buckling, as inferred from the analysis of changes in boundary conditions.
- The study agreed with prior findings about the influence of buckling mode and critical buckling load on a spherical shell under external pressure, providing further support for existing knowledge in this field.
- The obtained results collectively provide valuable insights into the structural behavior of imperfect spherical shells and have implications for the design and analysis of such structures.

Based on the results obtained from this study, here are some recommendations for future research:

- Investigating various imperfection types like ellipticity and out-of-roundness on the critical buckling load and deformation behavior of spherical shells could enhance our comprehensive understanding of imperfection behaviors.
- Incorporating the advanced material properties, including nonlinear material behavior, anisotropic materials, or composite materials, will influence the buckling behavior of imperfect spherical shells under external pressure.
- Conducting dynamic buckling analysis to examine the behavior of imperfect spherical shells under dynamic loading conditions, also could provide insights into the shell's stability under varying loading rates and frequencies.
- Conducting optimization studies to determine the optimal design parameters that can enhance the buckling resistance of imperfect spherical shells while considering factors such as material selection, thickness distribution, and boundary conditions.

By focusing on these areas for future research, it is possible to deepen the understanding of imperfect spherical shells' behavior under external pressure and develop more robust design approaches for such structures.

6. Acknowledgement

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Ilyani Akmar Abu Bakar and Jamaluddin Mahmud were involved in conceptualizing the problem, designed the research methods and worked on the whole framework of this study. While Muhamad Zarul Imani Mohd Zulkifli and Mohd Shahrom Ismail mostly conducted experimental data preparation. All authors contributed to the writing and review of the manuscript.

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