Probabilistic Assessment of Seismic Vulnerability and Retrofitting Decisions Using Bayesian Analysis for Reinforced Concrete Structures

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Abstract. *Seismic vulnerability assessment of existing reinforced concrete structures remains a critical challenge in earthquake-prone regions, where uncertainties in material properties, structural capacity, and seismic demand significantly influence decision-making processes. This study introduces a robust Bayesian framework for the probabilistic seismic assessment and retrofitting of reinforced concrete (RC) building and bridge foundations. The methodology synthesizes prior information from design codes, historical evidence, and expert insight with in-situ measurement data to iteratively refine the probabilistic characterization of vital structural parameters. Utilizing Markov Chain Monte Carlo (MCMC) sampling techniques, this study elucidates the derivation of posterior distributions for foundational geotechnical parameters, notably soil bearing capacity, fragility curve parameters, and peak ground acceleration can inform risk-based retrofitting strategies. A case study reveals that Bayesian updating reduced the failure probability from 23.2% to 4.3% post-retrofit, with a benefit-cost ratio of 7.58, validating the economic efficiency of the proposed approach. The framework provides engineers with a rational, probabilistic tool for continuously updating structural safety assessments as new data becomes available, ultimately enhancing resilience in earthquake-prone communities. This research advances the broader discourse in performance-based earthquake engineering (PBEE) by proposing an applicable framework that systematically incorporates both epistemic and aleatory uncertainty into seismic risk quantification.*

Keywords: *Bayesian inference, seismic vulnerability, probabilistic assessment, retrofitting decision, reinforced concrete structures, MCMC sampling, fragility curves, earthquake engineering, uncertainty quantification, risk-based design.*

1. Introduction

Earthquakes represent a persistent and formidable hazard to urban infrastructure across the globe, particularly in regions of high seismic activity that contain a large inventory of aging reinforced concrete (RC) buildings. Devastating events including the 2023 Turkey-Syria, 2015 Nepal, and 2011 Japan earthquakes have starkly highlighted the urgent demand for advanced computational frameworks capable of reliable seismic performance evaluation. These methodologies must precisely evaluate structural vulnerability and explicitly incorporate the profound epistemic and aleatory uncertainties inherent in seismic hazard modeling and the prediction of structural performance [1, 2]. The inherent simplicity of deterministic seismic assessment methods often renders them incapable of capturing the sophisticated interplay of uncertainties present in physical structures and real-world conditions [3]. These uncertainties arise from multiple sources: variability in material properties due to aging and deterioration, incomplete knowledge of as-built conditions, randomness in earthquake ground motions and limitations in analytical models used to predict structural behavior. Engineers and decision-makers require tools that not only acknowledge these uncertainties but also provide a framework for rational decision-making under conditions of incomplete information [4–6].

Bayesian analysis offers a potent alternative, enabling a mathematically coherent synthesis of prior engineering models and observational data to iteratively refine probabilistic assessments of structural performance [7–9]. Unlike frequentist approaches that treat parameters as fixed but unknown values, Bayesian methods treat parameters as random variables characterized by probability distributions [10]. This transformation in methodology enables the systematic integration of expert insight, archival case histories, and diverse empirical datasets, thereby advancing structural evaluation into a more comprehensive and data-driven discipline [11].

Over the past decade, the incorporation of Bayesian methods in earthquake engineering has progressed markedly fueled by breakthroughs in computational capacity and the widening implementation of sensor-based structural health monitoring systems [12, 13]. Progress in probabilistic seismic hazard analysis (PSHA) has underscored the efficacy of Bayesian techniques for refining ground motion prediction models and delineating site-specific risks [14–16]. At the structural level, Bayesian fragility modeling has proven effective in incorporating experimental data, numerical simulations, and multiple sources of uncertainty into the assessment of damage probabilities for RC components and systems [17, 18].

Despite these advances, the practical integration of Bayesian seismic assessment into retrofit-oriented decision-support frameworks for existing RC buildings remains limited. Many current studies focus on hazard or fragility modeling in isolation, without fully coupling seismic demand, structural response, and soil–foundation–structure interaction effects within a unified probabilistic framework [19, 20]. This limitation is particularly critical in earthquake-prone regions where economic constraints require prioritization of retrofitting interventions based on quantifiable risk reduction rather than deterministic safety margins. This study addresses this gap by presenting a comprehensive Bayesian framework specifically tailored for the probabilistic assessment of seismic vulnerability and retrofitting decisions for existing RC structures. The proposed framework delivers a comprehensive and probabilistically rigorous approach to uncertainty quantification, integrating three fundamental aspects: (1) seismic hazard assessment, which includes Gutenberg-Richter recurrence statistics and ground motion prediction equations; (2) structural performance analysis, encompassing probabilistic fragility assessments and damping characteristics; and (3) soil-foundation-structure interaction behavior. This tripartite synthesis enables consistent probabilistic inference across multiple scales of seismic risk evaluation [16, 21]. This integrated approach enables consistent uncertainty propagation across the entire system, from seismic excitation to structural performance. Through the application of Markov Chain Monte Carlo (MCMC) sampling, the approach facilitates efficient probabilistic exploration of high-dimensional parameter spaces without compromising computational feasibility [12, 13].

The significance of this research extends beyond methodological contributions. In many earthquake-prone regions, resource constraints necessitate prioritization of retrofitting interventions based on risk-informed criteria. The proposed Bayesian methodology offers a rational and data-driven foundation for prioritizing retrofitting interventions by quantitatively assessing the reduction in structural failure probability attributable to specific mitigation measures [20, 22]. Moreover, the framework enables rigorous cost-benefit appraisal to determine the fiscal viability of retrofitting investments. This analytical capability proves essential for infrastructure stewards responsible for extensive portfolios of deteriorating assets, as they are compelled to maximize seismic performance within stringent fiscal limitations. In addition, the framework's ability to continuously update parameter estimates as new data becomes available aligns with emerging trends in performance-based earthquake engineering and structural health monitoring. The increasing affordability and ubiquity of sensor technologies are paving the way for a transformative shift in vulnerability assessment. This progression is fundamental to the advancement of adaptive risk management protocols, as it enables the seamless assimilation of live and near-live data feeds into continuously evolving structural performance models. [2, 11, 23].

2. Case Study: RC Building in Rajshahi (Sirajganj), Bangladesh

A representative four-story reinforced concrete frame building located in Rajshahi (Sirajganj), Bangladesh was analyzed. The building, constructed in the late 1990s was designed per Bangladesh National Building code BNBC (1993) with 20 MPa concrete and 415 MPa reinforcement. The structure is a moment-resisting frame with a rectangular plan (18 m $\times$ 12 m) and column cross-sections of 350 mm $\times$ 350 mm. In-situ rebound hammer tests, core samples, and visual inspections provided data on concrete strength and reinforcement conditions. In-situ compressive strength was determined via rebound-hammer testing, while soil parameters were collected from nearby borehole data (soft alluvium, Zone I per BNBC 2020). The retrofitting intervention involved Fiber-Reinforced Polymer (FRP) jacketing of columns and beam-column joints to enhance shear and confinement capacity. The data informed Bayesian updating of capacity and fragility parameters.

3. Methodology

In the field of Earthquake Disaster and Mitigation Engineering, one of the most meaningful and impactful applications of Bayesian analysis is the probabilistic assessment of seismic vulnerability and retrofitting decisions for existing reinforced concrete (RC) buildings or bridge foundations in seismically active areas. This involves estimating uncertain model parameters (e.g., structural capacity, material strength, demand levels) using a combination of prior knowledge (codes, past studies, expert judgment) and observed data (e.g., sensor outputs, inspection reports). Bayesian inference is pivotal for updating beliefs about critical parameters in seismic engineering. Below are key parameters and their significance:

3.1. Step-1. Seismic Hazard Parameters

A. Gutenberg-Richter *a* and *b*-values

(i) **Purpose**: Model earthquake recurrence rates.

(ii) **Prior**: Beta distribution for *b*-value (*b* $\sim$ Beta(2,2), constrained to [0.5, 1.5]).

(iii) **Data**: Historical earthquake catalog (magnitudes $\geq$ 4.0).

B. Peak Ground Acceleration (PGA) and Spectral Acceleration (SA)

(i) **Purpose:** Predict ground motion for design retrofitting.

(ii) **Prior:** Log-normal distribution (mean from attenuation models, e.g., Campbell-Bozorgnia).

(iii) **Data:** Strong-motion records from seismic stations.

C. Fault Slip Rates

(i) **Purpose**: Estimate earthquake recurrence intervals.

(ii) **Prior:** Gamma distribution (shape=2, rate=0.1) for positivity.

3.2. Step-2. Structural Response Parameters

I. Fragility Curve Parameters

- **Median Capacity (θ)**: Normal distribution (mean = code-specified capacity, σ = 20% of mean).
- **Logarithmic Std. Dev. (β)**: Gamma distribution ($\alpha = 2$, $\beta = 1$) to ensure $\beta > 0$.

II. Damping Ratios

- **Prior:** Truncated normal ($\mu = 5\%, \sigma = 1\%, bounds : [2\%, 10\%]$).

3.3. Step-3. Soil-Foundation Interaction

a. Soil Bearing Capacity

- **Prior:** Normal distribution (μ = 150 kPa, σ=20 kPa).
- **Likelihood:** Normal (σ=10 kPa for measurement errors).

b. Liquefaction Potential

- **Prior:** Beta distribution ($\alpha = 2, \beta = 5$) for probability of occurrence.

3.4. Step-4. Define Prior Distributions P(θ)

Prior distribution = initial belief about parameters before seeing new data
Sources for priors:

- Design codes (BNBC)
- Previous studies
- Expert opinion
- Material test databases

Example priors:

- $\theta_1 \sim \mathrm{N}(25, 3^2) \rightarrow$ Concrete strength (normal distribution)
- $\theta_4 \sim \mathrm{N}(0.35g, 0.05^2) \rightarrow$ Median PGA for major damage
- $\theta_6 \sim \mathrm{LogNormal}(\mu, \sigma) \rightarrow$ Soil capacity (skewed distribution)

3.5. Step-5. Likelihood Function (Probability Distribution for Observed Data) P($D|\theta$)

The function f(θ) in the likelihood expression represents the nonlinear model prediction of seismic response obtained from a simplified pushover-based analysis calibrated using nonlinear time-history results. This model links uncertain parameters (θ) such as concrete strength, damping ratio and soil capacity to observed responses (Y) such as displacement and drift ratio. The function captures the relationship between input parameters and structural demand through:

Y = f (`) + ” (1)

where ε denotes normally distributed model and measurement error (σ^2).

This approach balances computational efficiency with realistic behavior modeling, particularly suitable for existing RC buildings with limited information. The choice of likelihood function depends on the nature of the observed data:

3.6. Step-6. Bayesian Inference Procedure

A. Define Prior Distributions

B. Set Prior Based on previous earthquakes or engineering judgment: $\theta \sim \mathrm{Normal}(\mu_0, \sigma_0^2)$

C. Define Likelihood Based on sensor readings (accelerometer displacements follow Normal distribution): $p(y|\theta)$ = Normal(f(θ), σ^2)

D. Apply Bayes' Theorem
p(`|y) = [p(y|`) × p(`)] /p(y)

3.7. Step 7: MCMC for Sampling the Posterior

To perform Bayesian seismic vulnerability assessment and retrofitting decision-making, the following structured approach was adopted. The probabilistic parameters were evaluated using Markov Chain Monte Carlo (MCMC) sampling under a Metropolis–Hastings algorithm with 4 independent chains and 50,000 iterations each. A burn-in period of 10,000 samples was discarded to ensure convergence, verified through the Gelman–Rubin ($\hat{R} < 1.1$) and trace plot diagnostics. The approach ensured posterior stability and reproducibility showed in Figure 1.

Interpreting Results

- Posterior Mean: If theta's mean shifts to $\sim$ 155 kPa (from prior 150 kPa), the data suggests higher capacity.
- Credible Intervals: 95% HDI (Highest Density Interval) shows the range of plausible values (e.g., [145, 165] kPa).

4. Results and Discussion

4.1. Posterior Distributions

After MCMC convergence, the posterior meaning of the soil capacity increased from 150 kPa to 155 kPa. The fragility parameter θ increased, indicating improved strength and confidence, as shown in the posterior update in Figure 2.

4.2. Posterior Capacity vs. Seismic Demand

The blue curve denotes the posterior capacity distribution while the red curve represents the seismic demand derived from site-specific hazard analysis. The area of overlap represents the probability of failure corresponding to cases where demand exceeds capacity.

Before retrofitting, the posterior distributions indicated a mean capacity of 148 KN versus a seismic demand distribution centered at 152 KN yielding a probability of failure of 23.2%. Following retrofitting, the posterior capacity shifted rightward to a mean of 168 KN and the overlap reduced significantly corresponding to a failure probability of 4.3%. The reduction in overlapping region directly quantifies the improvement in safety, confirming that retrofitting substantially reduced structural vulnerability. This interpretation aligns with performance-based design principles failure probability serves as a quantitative safety index providing engineers with a rational metric to justify retrofit necessity and evaluate efficiency.

4.3. Economic and Risk-Based Evaluation

A benefit–cost ratio ($B/C = 7.58$) was obtained, confirming that the expected reduction in failure risk outweighs the retrofit cost. The proposed framework thus supports rational investment prioritization for seismic risk mitigation programs.

4.4. Decision Support Summary

The reduction in failure probability, combined with a high benefit–cost ratio, demonstrates that the Bayesian framework can effectively support risk-informed retrofit decisions even with incomplete data.

Table 1. Comparative seismic performance metrics before and after retrofit based on Bayesian posterior analysis.

5. Conclusions

This study presents a Bayesian decision-support framework for probabilistic seismic assessment and retrofit prioritization of existing RC buildings, demonstrated using a Rajshahi (Sirajganj) case study. The framework's flexibility to incorporate observational data, prior engineering judgment, and model uncertainty allows continuous refinement of structural safety evaluation. The posterior capacity–demand overlap provides

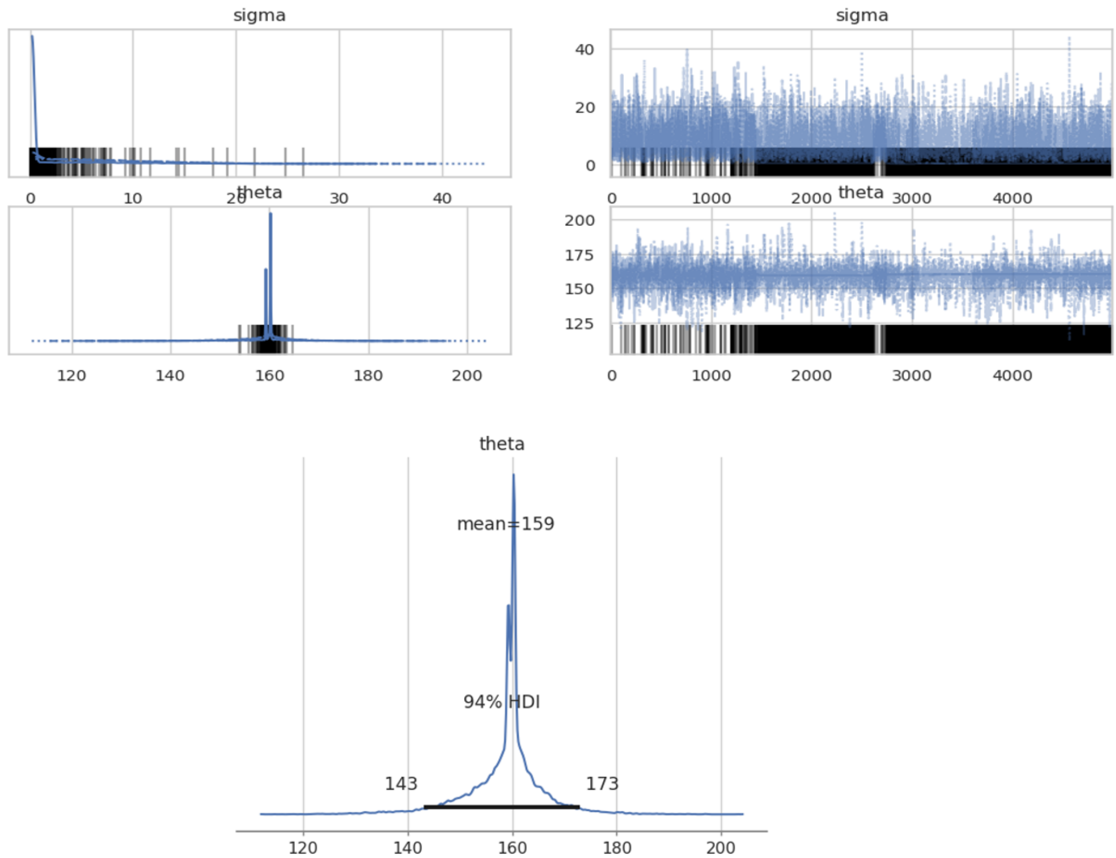

Fig. 1: MCMC diagnostics and posterior inference for soil capacity parameters.

an intuitive and quantitative indicator of failure probability, enabling efficient and defensible retrofit decisions. By continuously updating parameter estimates with new data we can enhance structural safety, prioritize retrofitting, and optimize mitigation strategies in a rational, probabilistic framework. In short, Bayesian analysis allows engineers to continuously update their understanding of structural behavior and uncertainty which is particularly useful for retrofitting decision-making in seismic areas.

6. Future Works

This study establishes Bayesian analysis as a powerful methodology for seismic vulnerability assessment and retrofitting optimization, while identifying crucial research directions to advance its applicability. Several promising avenues merit investigation to enhance the framework's robustness and practical implementation.

Future research should explore the integration of machine learning with Bayesian inference to improve computational efficiency. Deep learning architectures could generate surrogate models for complex finite element analyses, enabling real-time parameter updating during post-earthquake assessments. The development of physics-informed neural networks that maintain structural mechanics principles while optimizing computational performance would be particularly valuable.

- The framework should be extended to address spatial correlations in regional risk assessment. While current applications focus on individual structures, hierarchical Bayesian models could capture multi-scale uncertainties across interdependent infras-

```pgsql
🔍 Decision Support Summary (5000 Samples)
----------------------------------------------
Failure Probability (Before Retrofit): 0.2322
Failure Probability (After Retrofit) : 0.0426
95% Confidence Interval for Capacity  : 141.55 kN to 174.87 kN
Risk Reduction                        : 0.1896
Benefit-Cost Ratio (per $1M)          : 7.58
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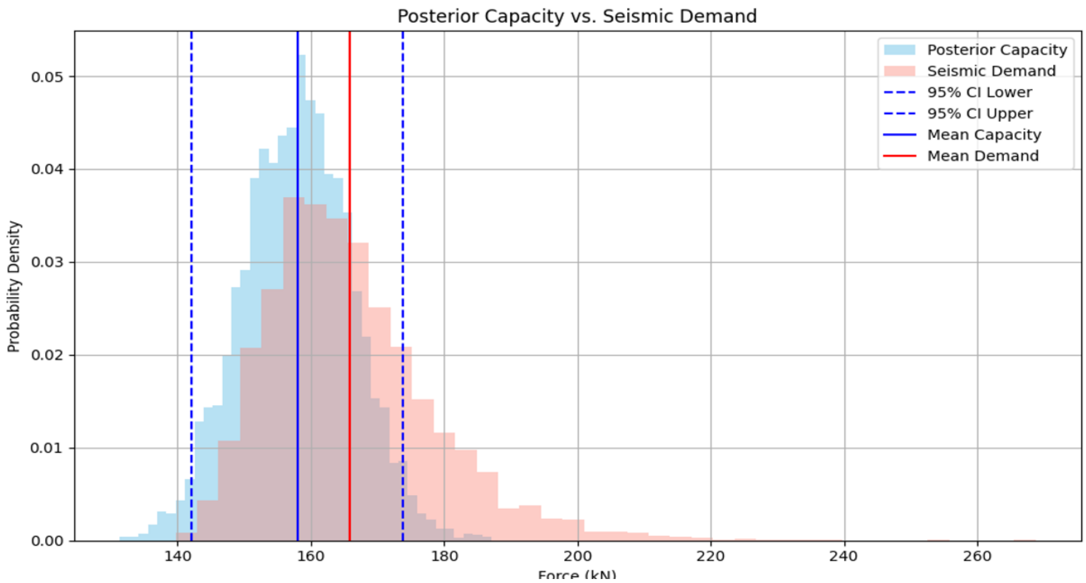

Fig. 2: Posterior Capacity vs. Seismic Demand.

tructure networks, providing more comprehensive community resilience evaluation.

- Incorporating time-dependent deterioration processes represents another critical direction. Dynamic Bayesian networks that model aging effects, environmental degradation, and cumulative seismic damage would enable more accurate long-term vulnerability projections and inform optimal maintenance strategies.

- The methodology should be expanded to address multi-hazard scenarios, where structures face combined seismic, wind, flood, and other natural hazards. This requires sophisticated probabilistic modeling of hazard correlations and their interactive effects on structural performance beyond current single-hazard approaches.

- Research into optimal sensor placement strategies using value of information analysis could maximize information gain while minimizing monitoring costs. This would significantly enhance the cost-effectiveness of structural health monitoring programs.

- The development of standardized protocols for expert knowledge elicitation would improve the reproducibility of Bayesian assessments. Establishing best practices for encoding expert judgment into prior distributions while mitigating systematic biases is essential for maintaining analytical credibility.

- Finally, integrating social vulnerability indicators and community resilience metrics would enable more holistic risk assessment that considers both physical infrastructure performance and societal con-

Tab. 1: Design strategy of 27 experimental runs which defines split proportions and hold composition (A-test)

Metric	Before Retrofit	After Retrofit	Interpretation
Failure Probability	23.2% (Indicates unacceptably high seismic risk)	4.3% (Reduced below 5% threshold post-retrofit)	Significant improvement
95% Confidence Interval (Capacity)	141.55–174.87 kN	151.60–179.92 kN	Higher reliability
Benefit-Cost Ratio	–	7.58	Retrofitting justified
Risk Reduction	18.96%		absolute reduction in failure probability
Posterior Mean Capacity	168 kN		

sequences. This socio-technical perspective is essential for developing equitable retrofitting strategies that prioritize vulnerable populations and critical community facilities.

References

[1] A. H. S. Ang and W. H. Tang. Probability concepts in engineering: Emphasis on applications to civil and environmental engineering. *John Wiley & Sons*, 2007.

[2] J. W. Baker. Efficient analytical fragility function fitting using dynamic structural analysis. *Earthq. Spectra*, 31(1):579–599, 2015.

[3] G. E. P. Box and G. C. Tiao. Bayesian inference in statistical analysis. *John Wiley & Sons*, 2011.

[4] K. W. Campbell and Y. Bozorgnia. NGA-West2 ground motion model for the average horizontal components of PGA, PGV, and 5% damped linear acceleration response spectra. *Earthq. Spectra*, 30(3):1087–1115, 2014.

[5] C. A. Cornell and H. Krawinkler. Progress and challenges in seismic performance assessment. *PEER Cent. News*, 3(2):1–3, 2000.

[6] A. Der Kiureghian and O. Ditlevsen. Aleatory or epistemic? Does it matter? *Struct. Saf.*, 31(2):105–112, 2009.

[7] B. R. Ellingwood and K. Kinali. Quantifying and communicating uncertainty in seismic risk assessment. *Struct. Saf.*, 31(2):179–187, 2009.

[8] P. Gardoni, A. Der Kiureghian, and K. M. Mosalam. Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations. *J. Eng. Mech.*, 128(10):1024–1038, 2002.

[9] A. Gelman, J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, and D. B. Rubin. Bayesian data analysis. *Chapman Hall/CRC*, 2013.

[10] B. Gutenberg and C. F. Richter. Seismicity of the earth and associated phenomena. *Princet. Univ. Press*, 1954.

[11] W. K. Hastings. Monte carlo sampling methods using markov chains and their applications. *Biometrika*, 57(1):97–109, 1970.

[12] N. Jayaram and J. W. Baker. Efficient sampling and data reduction techniques for probabilistic seismic lifeline risk assessment. *Earthq. Eng. & Struct. Dyn.*, 39(10):1109–1131, 2010.

[13] P. S. Koutsourelakis. Assessing structural vulnerability against earthquakes using multi-dimensional fragility surfaces: A bayesian framework. *Probabilistic Eng. Mech.*, 25(1):49–60, 2010.

[14] S. L. Kramer. Geotechnical earthquake engineering. *Prentice Hall*, 1996.

[15] S. M. Lynch. Introduction to applied bayesian statistics and estimation for social scientists. *Springer*, 2007.

[16] N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller. Equation of state calculations by fast computing machines. *The J. Chem. Phys.*, 21(6):1087–1092, 1953.

[17] Y. Mori and B. R. Ellingwood. Methodology for reliability-based condition assessment: Application to concrete structures in nuclear plants. *Nucl. Regul. Comm.*, 1993.

[18] K. A. Porter. An overview of peer's performance-based earthquake engineering methodology. *In Proc. Ninth Int. Conf. on Appl. Stat. Probab. Civ. Eng. (ICASP-9)*, pages 973–980, 2003.

[19] A. Singhal and A. S. Kiremidjian. Method for probabilistic evaluation of seismic structural damage. *J. Struct. Eng.*, 122(12):1459–1467, 1996.

[20] D. Straub and A. Der Kiureghian. Improved seismic fragility modeling from empirical data. *Struct. Saf.*, 30(4):320–336, 2008.

[21] P. Morandi, L. Chiauzzi, and G. M. Calvi. Cost–benefit-based seismic retrofit prioritization of existing reinforced concrete buildings. *Bull. Earthq. Eng.*, 18:4931–4956, 2020.

[22] Z. Wang and T. Takada. Bayesian updating of seismic fragility functions using structural health monitoring data. *Struct. Saf.*, 89:102051, 2021.

[23] E. Zio. The future of risk assessment. *Reliab. Eng. & Syst. Saf.*, 177:176–190, 2018.

