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Starfish Optimization, Sand Cat Swarm Optimization, Weighted Average and Mirage Search Optimization Algorithms for Finding Global Optimal Solutions

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Abstract. *In this study, the performance of four contemporary meta-heuristic techniques—namely Starfish Optimization, Sandcat Swarm Optimization, Weighted Average Algorithm (WAA), and Mirage Search Optimization—is investigated across diverse optimization challenges. The central goal is to execute a rigorous, unbiased comparative analysis to identify the most proficient optimizer among them. To ensure a fair benchmark, each method was configured with identical population sizes and iteration limits across five distinct problems. Furthermore, to guarantee statistical reliability, 50 independent trials were conducted for every algorithm. The results achieved by the four applied algorithms are compared to each other using different criteria, including the Minimum Fitness, Average Fitness, Maximum Fitness, and Standard Deviation. The comparison of the four criteria's values across the four algorithms reveals that WAA completely outperforms the others in all aspects, especially in the minimum fitness value and the standard deviation index. Moreover, WAA demonstrates superior stability and faster convergence during the search phase. Consequently, WAA is identified as the most effective solution and is highly endorsed for addressing the optimization tasks explored in this work.*

Keywords: *Sand Cat swarm optimization, Starfish optimization algorithm, Mirage search optimization, Weighted Average Algorithm, fitness functions; performance analyses.*

1. Introduction

Optimization problems are now appearing more frequently across fields such as economics and engineering. The primary goal in solving these problems is achieving the optimal solution—the minimum or maximum value of the objective function while satisfying all involved constraints, because such a solution yields significant benefits from both engineering and economic perspectives. Historically, this quest led to the development of traditional optimization methods. Specifically, the first and foremost is the gradient-based optimization methods mentioned in [1], which still demonstrate their advantage in solving several optimization problems listed in [2]. The second is Lagrange-based optimization methods, whose application is described in [3]. These Lagrange-based methods have been improved and modified for better efficiency, as shown in [4]. Moreover, the third is the Newton-Raphson-based optimization methods with a variety of applications described in [5]. Similar to the first two methods, as mentioned earlier, the Newton-Raphson-based optimization method was also improved and modified to achieve a higher level of efficiency [6]. However, in contemporary practice, these older techniques often struggle with large-scale and complex optimization problems. The presence of meta-heuristic algorithms soon redefined the term "optimization" in terms of application, solution determination, and the degree of optimization achieved. They effectively overcome the downsides of their predecessors, offering higher efficiency and faster response times when dealing with intricate problem landscapes. By using these modern techniques, high-quality solutions can be obtained reliably, resulting in a much higher success rate in solving today's most challenging optimization tasks.

Thanks to their simplicity in application, reliability, and robustness in search performance when dealing with a wide range of optimization problems, the applications of different meta-heuristic algorithms can be observed in various fields, including both economics and engineering, as mentioned and evaluated in [7]. Over time, many novel meta-heuristic algorithms with improved search performance have been continuously developed and introduced to serve the need of achieving higher optimization results. The novel meta-heuristic algorithms and their applications can be listed such as Five phases algorithm (FPA) [8], The Improved Dung beetle optimization (IDBO) [9], Enhanced Binary Kepler Optimization Algorithm (EBKO) [10], Conscious Neighborhood-Based Jellyfish Search Optimizer (CNJSO) [11], Enhanced Frilled Lizard Optimizer (EFLO) [12], Chaotic Harris Hawks algorithm (CHHA) [13], One-to-One Optimization Algorithm (OOA) [14], Krill Herd Algorithm (KHA) [15], Football Optimization Algorithm (FbOA) [16], Grey Wolf Optimizer (GWO) [17], Improved salp–swarm optimizer (ISSA) [18], Modified Adaptive Selection Cuckoo Search Algorithm (MASCSA) [19], Improved Whale Optimization Algorithm (IWOA) [20], Honey Badger Algorithm (HBA) [21], Osprey optimization algorithm (OOA) [22], Tornado optimizer (TO) [23], Multi-strategy Enhanced Snake Optimizer (MESO) [24], Skill Optimization Algorithm (SOA) [25], sine cosine algorithm (SCA) [26], Selforganizing migrating algorithm (SOMA) [27], Multi-Objective Genetic algorithm (MOGA) [28], dandelion optimization algorithm (DOA) [29], Ant colony optimization (ACO) [30], Hannibal Barca optimizer (HaBO) [31], hippopotamus optimization algorithm (HOA) [32], Bald Eagle Search Algorithm (BESA) [33], Snake Optimisation Algorithm (SnOA) [34], the Improved snow geese algorithm (ISGA) [35], marine predators algorithm (MPSA) [36], etc.

In this study, four recently proposed meta-heuristic algorithms are considered: the Starfish Optimization Algorithm (SFOA) [37], the Sandcat Swarm Optimization (SCSO) [38], the Weighted Average Algorithm (WAA) [39], and the Mirage Search Optimization [40]. While SFOA and MSO were both proposed in 2025, WAA and SCSO were first introduced in 2024 and late 2023, respectively. Additionally, SFOA and SCSO are developed based on mimicking the natural practices of starfish and sand cats, while WAA and MSO are formed through the simulation of physical phenomena. During the development phase, the four algorithms were tested with various test suites and demonstrated their capabilities, surpassing those of many previous ones in the comparison. This study is the very first paper to conduct a comparison among all these algorithms.

The main novelties and contributions of the whole study can be summarized as follows:

- Successfully applied four meta-heuristic algorithms, including the Starfish Optimization Algorithm (SFOA), the Sandcat Swarm Optimization (SCSO), the Weighted Average Algorithm (WAA), and the Mirage Search Optimization to determine the best fitness values for five different optimization problems.

- Offer a thorough evaluation and discussion about the searching performance of all four algorithms on each given optimization problem using different criteria.
- Reveal and indicate the best applied algorithm among the four in dealing with all the considered optimization problems.
- Provide a valuable reference in assessing and ranking the performance of particular meta-heuristic algorithms in dealing with certain optimization problems.

2. Problem Formulation

Optimization problems fundamentally consist of two critical elements. The first is the objective function, which is the mathematical relationship among all the variables. The second element is the set of constraints, which are the specific conditions, limitations, or rules that the variables must satisfy. Any viable solution must strictly adhere to these prescribed boundaries while simultaneously achieving the best possible result in accordance with the defined goal.

2.1. The main objective function

Normally, a typical objective function of a given optimization problem is described as follows:

$$OBF(y_1, y_2, \ldots, y_k) \tag{1}$$

where, OBF is the value of the given objective function; $y_1, y_2, \ldots, y_k$ are the variables that structure the mathematical model of the considered objective function, and k is the dimension of the considered optimization problem.

2.2. The constraints

The constraints that must be rigorously adhered to are a fundamental part of solving any optimization challenge, as previously discussed. These constraints are responsible for establishing clear boundaries for all variables. Typically, the constraints are categorized into two types: equality constraints and inequality constraints:

1) The equal constraints

The equal constraints are main foundation to calculate the dependent variables in the process of resolving any optimization problem. Suppose that y_1 is denoted as the dependent variable, and $y_2, y_3, \ldots, y_n$ are the control ones. The equal constraint is generally described as follows:

$$\varepsilon_1 y_1 + \varepsilon_2 y_2 + \varepsilon_3 y_3 + \ldots - \varepsilon_k y_k = \tau \tag{2}$$

where, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots, \varepsilon_k$, and ε are the available coefficients characterized by the given optimization problem; y is the dependent variable, τ is a constant values.

2) The inequality constraints

Typically, inequality constraints are leveraged by meta-heuristic algorithms to define the search space and generate the initial values for the control variables at the beginning of the optimization procedure. These constraints are often formulated as follows:

$$y_2^{\min} \le y_2 \le y_2^{\max} \tag{3}$$

$$y_3^{\min} \le y_3 \le y_3^{\max} \tag{4}$$

$$y_k^{\min} \le y_k \le y_k^{\max} \tag{5}$$

In Equations (3) – (5), $y_2^{\min}, y_3^{\min}, \ldots, y_k^{\min}$ are the lower boundaries of $y_2, y_3, \ldots, y_k$, and $y_2^{\max}, y_3^{\max}, \ldots, y_k^{\max}$ are their upper boundaries.

2.3. The considered optimization problem in the paper

1) The first optimization problem

The mathematical expressions of the main objective function and other related constraints of the first optimization problem are given as below [40]:

$$TF_1 = \sum_{k=1}^{dim} (y_k - 1)^2 \quad \text{with } k = 1, 2, \ldots, dim \tag{6}$$

And

$$y_{TF1}^{LB} \leq y_1, y_2, \ldots, y_k \leq y_{TF1}^{UB} \tag{7}$$

In Equations (6) and (7), TF_1 is the value of the first test function; k is the count of variables required to be determined for obtaining an optimal solution that corresponds to the best value of the primary objective function; dim is the dimension of the considered problem, which is set by 20; y_{TF1}^{LB} and y_{TF1}^{UB} are the lower and upper bounds of variables.

2) The second optimization problem

Similar to the initial problem, the second optimization problem is also characterized using the objective function and the relevant constraints via specific mathematical expressions as presented below [38]:

$$TF_2 = \sum_{k=1}^{dim} |y_k| + \prod_{k=1}^{dim} |y_k| \quad \text{with } k = 1, 2, \ldots, dim \tag{8}$$

And

$$y_{TF2}^{LB} \leq y_1, y_2, \ldots, y_k \leq y_{TF2}^{UB} \tag{9}$$

where, TF_2 is the value of the second test function; y_{TF2}^{LB} and y_{TF2}^{UB} are the lower and upper bounds of variables that are required to be determined for establishing an optimal solution.

3) The thrid opimization problem

In parallel with the two optimization problems mentioned above, the mathematical definition of the objective function and the variable constraints are provided as follows [37]:

$$TF_3(y, dim) = \sum_{k=1}^{dim} \left(y_k^2 - 10\cos(2\pi y_k)\right) + 10 \times dim \tag{10}$$

In Equation (10), TF_3 is the value of the third test function; dim is dimension of the search space; k is the index of the variable in the summation, ranging from 1 to dim: $k = 1, 2, \ldots, dim$.

4) The fourth optimization problem

Similar to the preceding case, the mathematical forms of the main objective function and the related constraint are provided below [40]:

$$TF_4(y) = -\sum_{k=1}^{7} \left[(y_k - a_{SHk})^2 + c_{SHk}\right]^{-1} \tag{11}$$

with $k = 1, 2, \ldots, 7$.

In Equation (11), TF_4 is the value of the fourth test function; k is the index of the variable in the summation, ranging from 1 to 7; c_{SHk} is the kth element of vector c_{SH}; a_{SHk} is the kth element of vector a_{SH}; Matrix a_{SH} is a 10×4 matrix containing the coefficients:

$$a_{SH} = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 5 & 3 & 3 \\ 8 & 1 & 8 & 1 \\ 6 & 2 & 6 & 2 \\ 7 & 3.6 & 7 & 3.6 \end{bmatrix}$$

And, vector c_{SH} is a 1×10 vector:

$$c_{SH} = [0.1\ 0.2\ 0.2\ 0.4\ 0.4\ 0.6\ 0.3\ 0.7\ 0.5\ 0.5] \tag{12}$$

5) The fifth optimization problem

Following the same formulation approach as above, the mathematical expressions of the main objective function and its associated constraint are given below [39]:

$$TF_5(y) = -\sum_{k=1}^{10} \left[(y_k - a_{SHk})^2 + c_{SHk}\right]^{-1} \tag{13}$$

where, TF_5 is the value of the fifth test function; k is the index of the variable, ranging from 1 to 10;

3. The Applied Algorithms

3.1. Starfish optimization algorithm

This section provides a brief description of the mathematical formulation underlying the Starfish Optimization Algorithm (SFOA). As a metaheuristic approach, SFOA follows a structural framework similar to other algorithms within this category. The distinctive feature of SFOA lies in its solution update mechanism, which is executed through two sequential stages described below.

1) Stage 1

In this first phase, the new solutions will be updated using the following model [37]:

$$S_i^{new1} = \begin{cases} \begin{cases} S_i + FFA \times (S_{top} - S_i) \times \cos\theta, \\ S_i + FFA \times (S_{top} - S_i) \times \sin\theta, \end{cases} \\ GF \times S_i + rgv_1 \times (S_{rand1} - S_i) \\ +rgv_2 \end{cases} \tag{14}$$

With

$$GF = \frac{L - l}{L} \times \cos\theta \tag{15}$$

In Equations (14) and (15), S_i^{new1} is the modified solution obtained during the first phase, with index i ranging from 1 to NS and NS is the total number of solutions in the population; S_i is solution i at the present state; FFA is the factor for amplification; S_{top} is the top-performing solution found at current; θ is the phase angle between the current solution and the optimal solution; GF is the guidance factor; rgv_1, rgv_2 and rgv are the randomly generated values between zero and one; S_{rand1} and S_{rand2} are two solutions chosen randomly from the existing population; L is the iteration limit; l is the iteration number at present.

2) Stage 2

In this section, the new solutions are updated using the following expression [37]:

$$S_i^{new2} = \begin{cases} S_i + rgv_1 \times D_1 + rgv_2 & \\ \times D_2, & \text{if } i \neq NS, \\ \exp\left(\frac{-L \times NS}{L}\right) \times S_i, & \text{if } i = NS \end{cases} \tag{16}$$

where, S_i^{new2} is the modified solution obtained during the second phase; D_1 and D_2 are the distances from two randomly selected solutions to the top-performing solution.

According to the author in [37], the entire optimizing process of SFOA is executed based on the following pseudo code:

If $rand > G_p$

Execute the update method using Equation (14),

Else

Execute the update method using Equation (16),

End.

3.2. Sand Cat Swarm algorithm

As stated earlier, the Sand Cat Swarm Optimization (SCSO) algorithm operates through two primary stages: exploration and exploitation. These stages constitute the core mechanism for updating candidate solutions throughout SCSO's iterative optimization procedure. The mathematical formulations governing these two stages are detailed as follows.

1) Stage 1

In this stage, the revised position of a candidate solution is obtained by multiplying the distance vector from its current coordinates to its personal best position by a guidance parameter, formulated as follows [38]:

$$S_i^{new1} = (S_{top} - rgv \times S_i) \times GF \qquad (17)$$

where S_i^{new1} is the modified solution obtained during the first phase, with index i ranging from 1 to NS and NS is the total number of solutions in the population; S_{top} is the top-performing solution found at current; rgv are the randomly generated values between zero and one; S_i is solution i at the present state; and GF is the guidance factor calculated using the following expression [38]:

$$GF = \alpha \times rgv \qquad (18)$$

With

$$\alpha = 2 - \left(\frac{4 \times l}{l + L}\right) \qquad (19)$$

where, α is the adjustment coefficient; L is the iteration limit; l is the iteration number at present.

2) Stage 2

In this phase, the new location of each individual in the population will be updated for its new location using the following mathematical model [38]:

$$S_i^{new2} = S_{gtop} - C_{rand} \times \cos(\theta) \times GF \qquad (20)$$

where S_i^{new2} is the modified solution obtained during the second phase; S_{gtop} is the top global best solution in the whole population; S_{rand} is the solution chosen randomly from the existing population; θ is the phase angle between the current solution and the optimal solution.

Similar to the SFOA, the whole optimizing process for optimal solution of the SCSO is executed using the same structure as follows:

If $|R| \leq 1$

Execute the update method using Equation (17),

Else

Execute the update method using Equation (20),

End

3.3. Weighted Average Algorithm

The Weighted Average Algorithm (WAA), proposed in 2024, is a metaheuristic optimization technique that computes the weighted average position of all individuals in the population and employs two alternative search mechanisms. The position update mechanism of WAA is designed to achieve a balanced trade-off between the exploration stage and the exploitation stage, which will be elaborated in the subsequent sections.

1) Stage 1

In this phase, WAA executes three different strategies subsequently to update the new solution as follows [39]:

$$\begin{aligned} S_i^{new1} = & \; rgv \times (S_{was} - S_{gtop}) \\ & + rgv \times (S_{was} - S_{BS,i}) \\ & + rgv \times S_{was} \end{aligned} \qquad (21)$$

$$\begin{aligned} S_i^{new2} = & \; rgv \times (S_{was} - S_{BS,i}) \\ & + rgv \times S_{BS,i} \end{aligned} \qquad (22)$$

$$\begin{aligned} S_i^{new3} = & \; rgv \times (S_{was} - S_{gtop}) \\ & + rgv \times S_{gtop} \end{aligned} \qquad (23)$$

where S_i^{new1}, S_i^{new2}, S_i^{new3} are, respectively, the ith updated solution obtained when the three strategies are sequentially applied with $i = 1, 2, \ldots, NS$; S_{was} is the weighted average solution; and $S_{BS,i}$ is the best solution found so far for individual i. S_{gtop} is the top global best solution in the whole population; rgv are the randomly generated values between zero and one.

2) Stage 2

In this stage, WAA implements the update mechanism by randomly selecting one of two available models as given in the following equation [39]:

$$S_i^{new} = \begin{cases} S_{gtop} \times MSF, & \text{if } rgv > 0.5, \\ rgv \times (\gamma_{max} - \gamma_{min}) & \\ +\gamma_{min}, & \text{else} \end{cases} \quad (24)$$

where, MSF is a multiplicative scaling factor derived from the Levy stable distribution; γ_{max} is the maximum permissible boundary value defining the upper constraint of the feasible solution region; γ_{min} is the minimum permissible boundary value defining the lower constraint of the feasible solution region.

The entire update process for new solution of WAA is performed by the following pseudo code:

If $r_1 < 0.5$

If $r_2 > 0.5$

Execute the update method using Equation (24),

Else

Switch r_3

- Case 1: Execute the update method using Equation (21).

- Case 2: Execute the update method using Equation (22).

- Case 3: Execute the update method using Equation (23).

End switch

End

End.

** Note that r_1 is the selecting factor; r_2 and r_3 are the random values between zero and one.*

3.4. Mirage search optimization

As previously discussed, the key characteristic that distinguishes Mirage Search Optimization (MSO) from other algorithms lies in its solution update mechanism. This updating process incorporates two distinct strategies: the generation of upper images and the generation of lower images. The mathematical formulations for these two strategies are presented in detail in the following subsections.

1) Stage 1

In this stage, all solutions in the initial population are updated according to the following model [40]:

$$S_i^{new1} = S_i + \Delta S_{ph1} \quad (25)$$

With

$$\Delta S_{ph1} = \frac{\sin\theta_1 \times DF \times \sin\theta_2}{\sin\theta_3 \times \sin\theta_4} \quad (26)$$

$$DF = |S_{top,i} - S_i| \times rgv_1 + 1 \quad (27)$$

In Equations (25) – (27), S_i^{new1} is the modified solution obtained during the first stage, with index i ranging from 1 to NS, and NS is the total number of solutions in the population; S_i is the solution i at the present stage; ΔS_{ph1} is the phase angle difference in the first stage; $S_{top,i}$ is the top-performing solution found at iteration i; $\sin\theta_1$ is the sine of the angle between the reflected ray and the horizontal reference line; $\sin\theta_2$ is the sine of the angle between the incident ray and the normal at the point of incidence; $\sin\theta_3$ is the sine of the angle between the refracted ray and the normal at the point of incidence; and $\sin\theta_4$ is the sine of the angle between the incident ray and the horizontal reference line; rgv_1 are the randomly generated values between zero and one; DF is the distance factor from the current solution to the best solution.

2) Stage 2

In this second stage, all the solutions will be updated by applying the following expression [40]:

$$S_i^{new2} = S_i + \omega_3 \Delta S_{ph2} \quad (28)$$

With

$$\Delta S_{ph2} = \frac{DF}{\tan\mu} \quad (29)$$

$$\mu = \frac{3.14 \times (L - l) \times rgv_2}{2 \times l} \tag{30}$$

Where, S_i^{new2} is the modified solution obtained during the second stage; ΔS_{ph2} is the phase angle difference in the second stage; μ is the phase angle between the refracted stratified line and the horizontal reference line; ω_3 is a coefficient that controls the step size in the second stage; rgv_2 are the randomly generated values between zero and one.

Unlike SFOA and SCSO, the update process for new solutions of MSO execute both phase as described above:

While $l < L$

▷ **Execute Stage 1**

- Step 1: Execute the update method for new solution using Equation (25).
- Step 2: Apply the boundary check for new solutions.
- Step 3: Calculate the fitness value for new solutions.
- Step 4: Determine the best solution.

End Stage 1.

▷ **Execute Stage 2**

- Step 5: Execute the update method for new solution using Equation (28).
- Step 6: Apply the boundary check for new solutions.
- Step 7: Calculate the fitness value for new solutions.
- Step 8: Determine the best solution.

End Stage 2.

End.

4. Results

In this section, four recent proposed meta-heuristic algorithms, including SFOA [37], SCSO [38], WAA [39], and MSO [40] will be applied to determine the best solution to five different benchmark functions derived from the CEC 2029 test suite. Then, the results obtained by the four applied algorithms will be analyzed and evaluated in detail to identify the strengths and weaknesses of each algorithm when dealing with the considered benchmark functions. For a fair comparison, the four algorithms are using the same presets regarding population size (NS) and maximum iteration index (L). Particularly, NS and L of the four applied algorithms are set to 20 and 100 while dealing with the five benchmark functions as mentioned earlier. Additionally, each applied algorithm is run for 50 test runs to determine the best solution before any comparison is made.

The entire study is conducted on a personal computer with the following basic specifications: a central processing unit with a 2.26 GHz clock speed and 16 GB of random-access memory (RAM). Furthermore, the involved coding and simulations are performed using MATLAB R2018a.

4.1. Results obtained on the first problem

Figure 1 shows the results from the 50 test runs clearly of the four applied algorithms, including SFOA, SCSO, WAA, and MSO for the first optimization problem as formulated in Section 2. . The WAA and MSO demonstrated significantly superior stability and efficiency, consistently converging to a near-zero fitness value across almost all trials, confirming their robustness. In stark contrast, the SCSO exhibited highly erratic behavior, frequently spiking to fitness values around 3, indicating a high failure rate in finding the global optimum. Worst of all was the SFOA, which produced the most extreme failures, with fitness values reaching 4. This evaluation confirms that WAA and MSO are the most reliable and effective algorithms, while SCSO and SFOA suffer from poor search balance, making them unreliable for this specific optimization task.

The convergence analysis across the minimum, mean, and maximum convergences in Figure 2 demonstrates a clear differentiation in

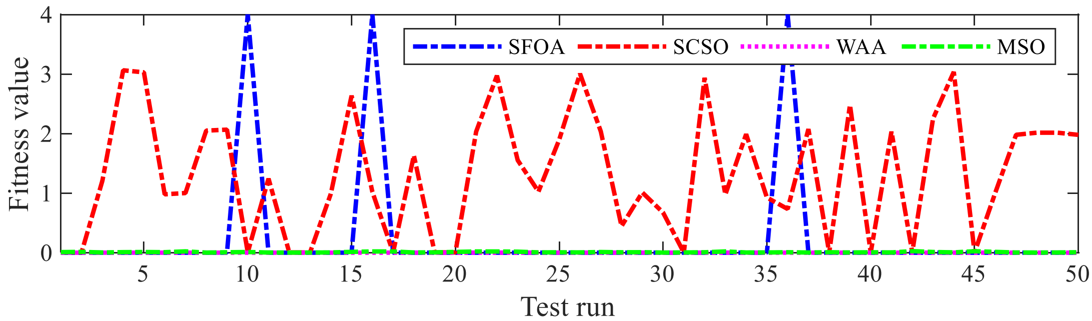

Figure 1: The fitness values obtained by the four applied algorithms after 50 test run while dealing with the frist optimization problem.

the search capabilities of the four algorithms. The WAA exhibited unrivaled speed and consistency, achieving optimal convergence to a near-zero fitness value in fewer than 10 iterations across all scenarios, indicating exceptional stability and reliable search performance. Similarly, the MSO proved highly effective, matching WAA's final solution quality, though converging slightly slower. In contrast, both the SCSO and the SFOA showed significantly impaired search performance; SCSO stagnated prematurely around a fitness of 3, while SFOA displayed the worst result, stagnating near a fitness of 4 in the mean and maximum trials. This stark performance gap confirms that WAA and MSO possess a superior exploration and exploitation balance necessary to locate the true global optimum for this problem consistently.

Table 1 shows the qualitative results achieved by the four applied algorithms across different criteria, including Minimum fitness (Min.Fit), Average fitness (Aver.Fit), Maximum fitness (Max.Fit), and Standard deviation (Std). The table clearly shows that SFOA and WAA are superior to the others; however, WAA is the most effective algorithm across all four criteria.

Table 1: The summarized results of the four algorithms dealing with the first optimization problem.

	SFOA	SCSO	WAA	MSO
Min.Fit	0.000	0.001	0.000	0.001
Ave.Fit	0.240	1.321	0.000	0.009
Max.Fit	4.000	3.063	0.001	0.026
Std	0.960	1.031	0.000	0.007

4.2. Results obtained on the second problem

In this section, the results achieved by the four applied algorithms while dealing with the second optimization problem, as shown in Section 2. , will be presented and analyzed. Figure 3 provides an overall analysis of the four algorithms across the test runs when applied to determine the optimal solution to the second optimization problem. In the figure, WAA still offers the most consistently high-performing optimizing capability. Particularly, WAA and MSO demonstrated superior robustness, quickly converging to the optimum, whereas SFOA and SCSO were highly unreliable, suffering from frequent, large-scale convergence failures and stagnation. However, when tackling the second, high-precision problem (high-zoom inset graph), SFOA, SCSO, and WAA all achieved near-perfect stability, with the fitness values located in the 10^{-11} range. In that case, MSO's performance degraded significantly, yielding only an acceptable fitness range of 0.1 to 0.3, confirming that while WAA is universally robust, the success of SFOA and SCSO is heavily contingent upon the specific characteristics of the optimization problem.

In Figure 4, the convergence achieved by the four applied algorithms are shown. At first, the graphs in the figure show a trade-off between speed and overall search effectiveness throughout all the subfigures with a) the minimum convergence, b) the mean convergence, and c) the maximum convergence. The WAA, SCSO,

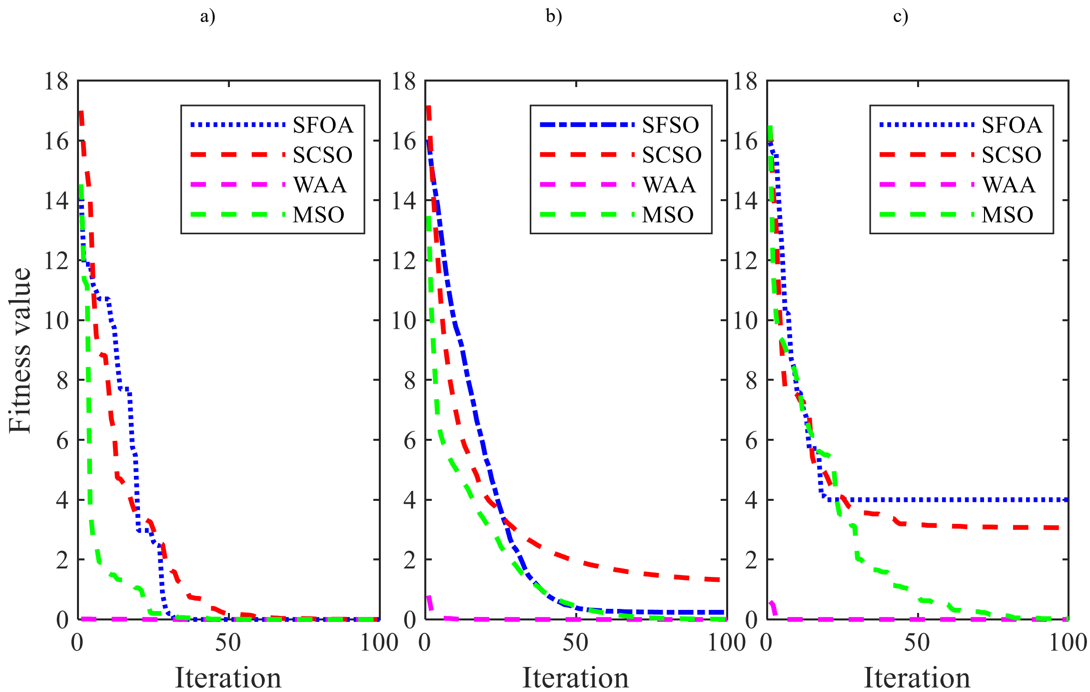

Figure 2: a) The minimum, b) mean, and c) maximum convergence obtained by the four applied algorithms for the best runs while dealing with the first optimization problem.

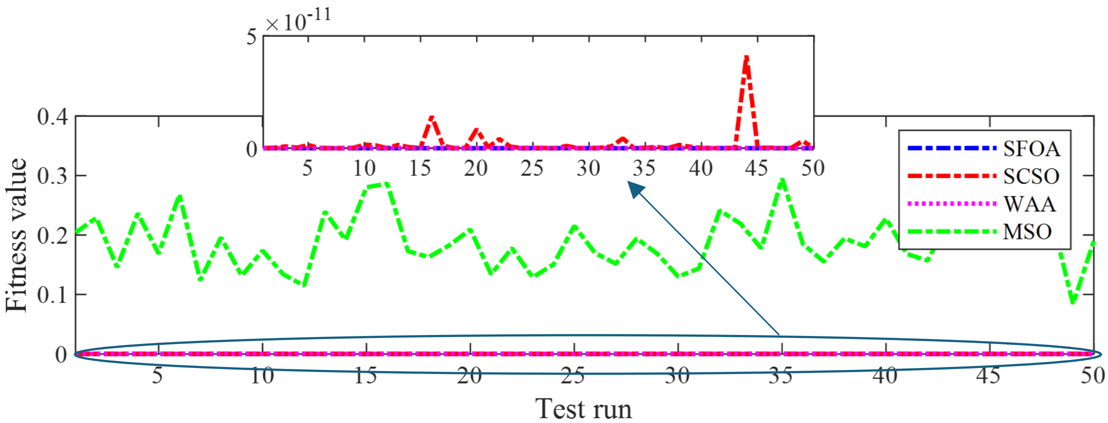

Figure 3: The fitness values obtained by the four applied algorithms after 50 test runs the second optimization problem.

and SFOA all demonstrated exceptionally fast convergence, achieving the optimal fitness value within the first 20 iterations. Conversely, the MSO was significantly slower, requiring the full 100 iterations to reach the same final solution. However, since the minimum, mean, and maximum convergence for WAA and MSO were all close to the final optimum, they exhibited high consistency in their search process. Meanwhile, the fast initial drop for SCSO and SFOA is misleading, as the separate reliability analysis (from the 50 trial runs) confirmed their fast search is highly erratic and prone to massive failures.

The detailed results on the four different criteria of the four applied algorithms while

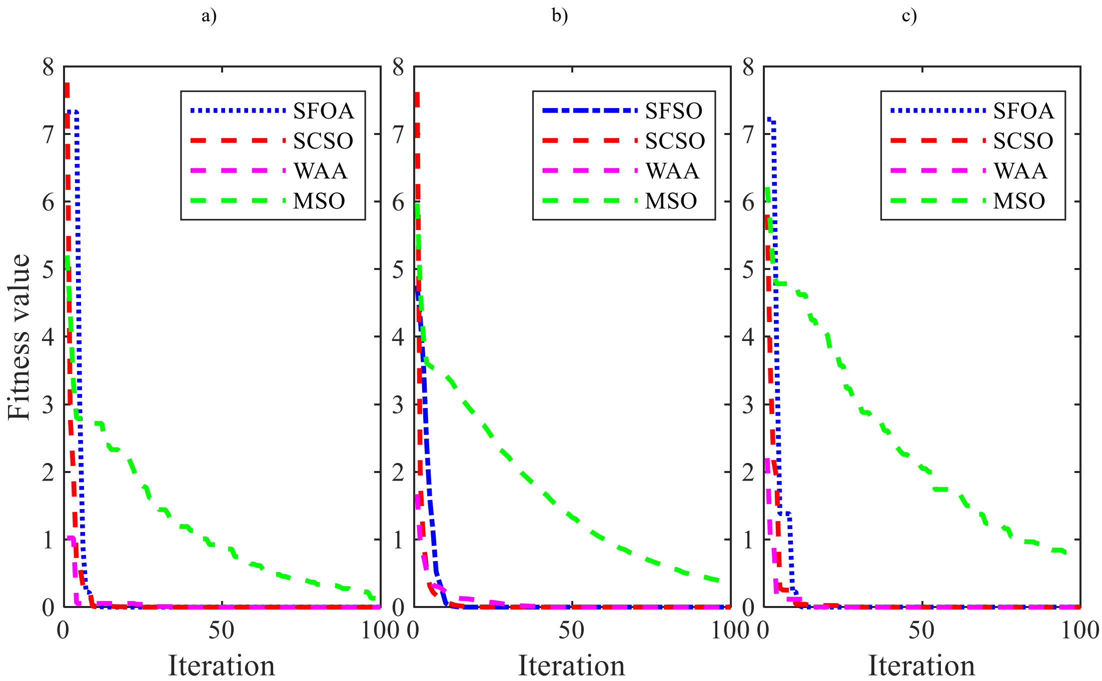

Figure 4: a) The minimum, b) mean, and c) maximum convergence obtained by the four applied algorithms for their best runs while dealing with the second optimization problem.

dealing with the second optimization problem are displayed in Table 2. According to the table's results, WAA remains the most effective across all four criteria.

4.3. Results obtained on the third problem

This section provides the performance analyses of the four applied algorithms in dealing with the third optimization problem, which is described in detail in Section 2. Figure 5 shows the fitness values obtained by the four algorithms after 50 trial tests. The observation in the figure indicates that the WAA, SCSO, and SFO all achieved perfect stability, consistently converging to a near-zero fitness value across every trial run, thereby confirming their suitability for this problem. Conversely, the MSO continuously proved itself entirely unreliable, failing to find the global optimum in any run and oscillating erratically between high fitness values (30 to 90) as shown in. This massive performance gap confirms that MSO is fundamentally ill-suited for the complex landscape of the third optimization problem, while the other three demonstrate high robustness.

Besides, the convergence analysis for the third problem of SFOA, SCSO, and WAA is also given in Figure 6, which demonstrates exceptionally fast convergence, reaching the optimal fitness value of the given optimization problem in under 20 iterations across the minimum, mean, and maximum convergence. This rapid convergence, coupled with the perfect reliability shown in the 50-run chart, establishes them as the most robust solvers. In contrast, the MSO's convergence in Figure 6 proves fundamentally flawed for this problem. At the same time, it starts quickly, but it soon becomes the slowest, least consistent algorithm, stagnating at high fitness values (60–90) in the mean and maximum convergence, which clearly explains its total failure across the 50 trial runs. Besides, the qualitative results obtained by the four applied algorithms are presented in Table 3. In the ta-

Table 2: The summarized results of the four algorithms dealing with the second optimization problem.

	SFOA	SCSO	WAA	MSO
Min.Fit	$1.2132E{-}274$	$3.04082E{-}16$	$3.6967E{-}45$	0.084
Ave.Fit	$2.9002E{-}178$	$1.84966E{-}12$	$6.19623E{-}33$	0.193
Max.Fit	$8.0137E{-}177$	$4.12707E{-}11$	$1.70895E{-}31$	0.367
Std	0.00213	$6.17431E{-}12$	$2.67892E{-}32$	0.055

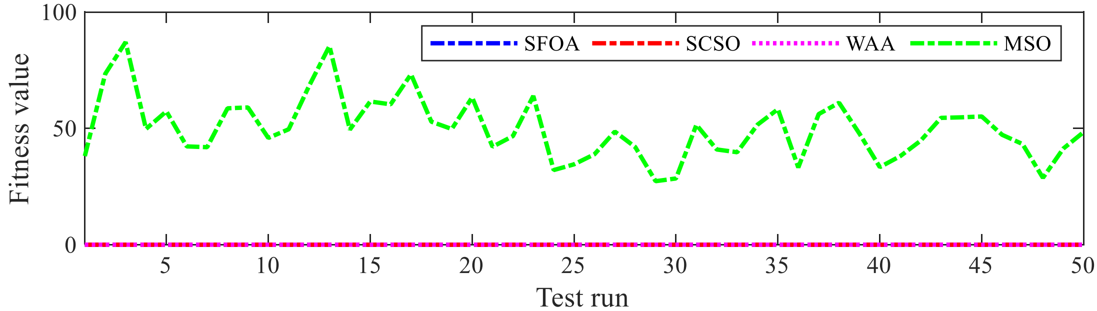

Figure 5: The fitness values obtained by the four applied algorithms after 50 test runs the thrid optimization problem.

ble, SFOA, SCSO, and WAA offer similar performance, while MSO is the worst method.

Table 3: The summarized results of the four algorithms dealing with the third optimization problem.

	SFOA	SCSO	WAA	MSO
Min.Fit	0.000	0.000	0.000	27.303
Ave.Fit	0.000	0.000	0.000	50.019
Max.Fit	0.000	0.000	0.000	87.071
Std	0.000	0.000	0.000	13.377

4.4. Results obtained on the fourth problem

This section provides a performance evaluation of the four applied algorithms in addressing the fourth optimization problem, highlighting the differences in the function compared to the previous one. The analysis of the fourth optimization problem, conducted over 50 test runs shown in Figure 7, establishes the WAA as the only fully reliable and robust optimizer, consistently achieving the true global minimum (approximately -10) in every single trial. In stark contrast, the SFOA, SCSO, and MSO are highly unreliable for this landscape. Crucially, these three algorithms exhibit chaotic and erratic search behavior, with their fitness values fluctuating widely across a range of nearly 10 units (from near -10 to 0), representing significantly higher and more damaging volatility compared to their performance in the less-complex previous problems. This confirms that the complexity of the fourth problem fundamentally compromises their ability to escape local optima consistently.

Figure 8 presents the three convergences, including a) the minimum, b) the mean, and c) the maximum convergence achieved by the four applied algorithms while solving the fourth optimization problem for their best runs. The observations from all the subfigures indicate that the WAA still maintains its superiority compared to others, offering exceptionally fast convergence and stability by reaching the optimal fitness value (near -10.3) within the first 10 iterations across all convergence. The MSO is the next best option, achieving a comparable optimal minimum but being significantly slower and exhibiting severe stagnation (at -5.8) in both its mean and maximum convergence. Conversely, SFOA and SCSO are the least effective, exhibiting slow, stepwise convergence and fail-

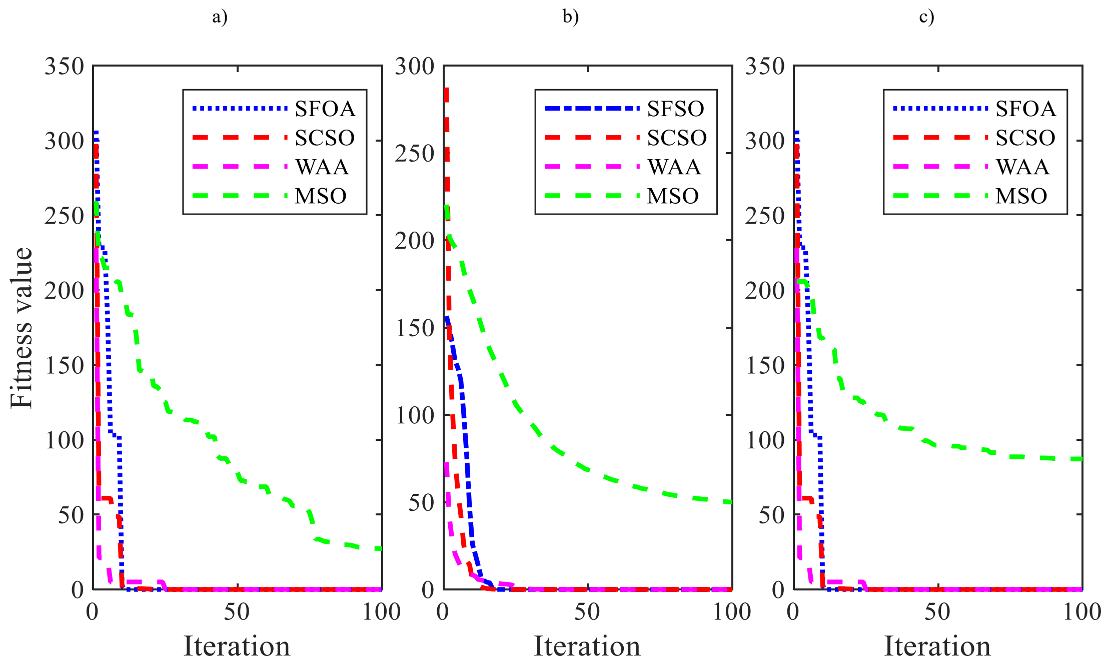

Figure 6: a) The minimum, b) mean, and c) maximum convergence obtained by the four applied algorithms for the best runs while dealing with the third optimization problem.

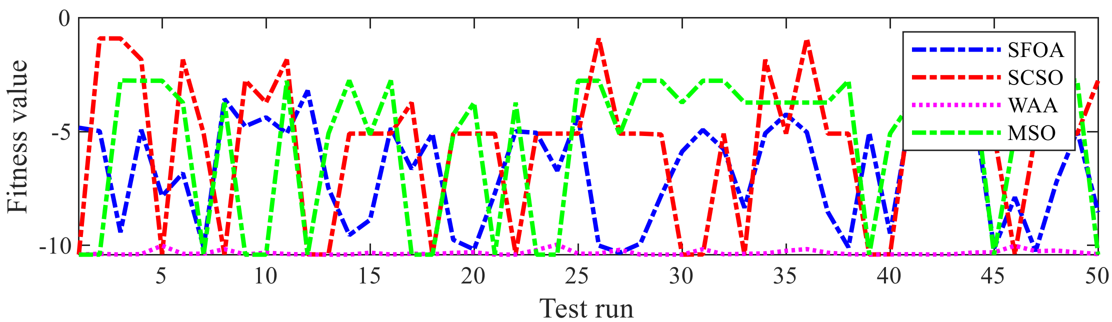

Figure 7: The fitness values obtained by the four applied algorithms after 50 test runs the fourth optimization problem.

ing to reach the optimum, instead settling at poor local optima (around -2.5) in their mean and maximum trials, which highlights their profound unsuitability for complex landscapes.

Table 4 clearly shows the certain values achieved by SFOA, SCSO, WAA, and MSO through the four criteria. Data in the table WAA again shows that it is the most effective algorithm among the four.

4.5. The Results obtained on the fifth problem

In this section, the evaluation of the fifth optimization problem, which represents a complexity improvement over the optimization presented in the previous section, will be provided. First, Figure 9 shows the fitness values achieved by the four algorithms over 50 test runs. The figure confirms that the WAA is the sole reliable performer, maintaining perfect stability

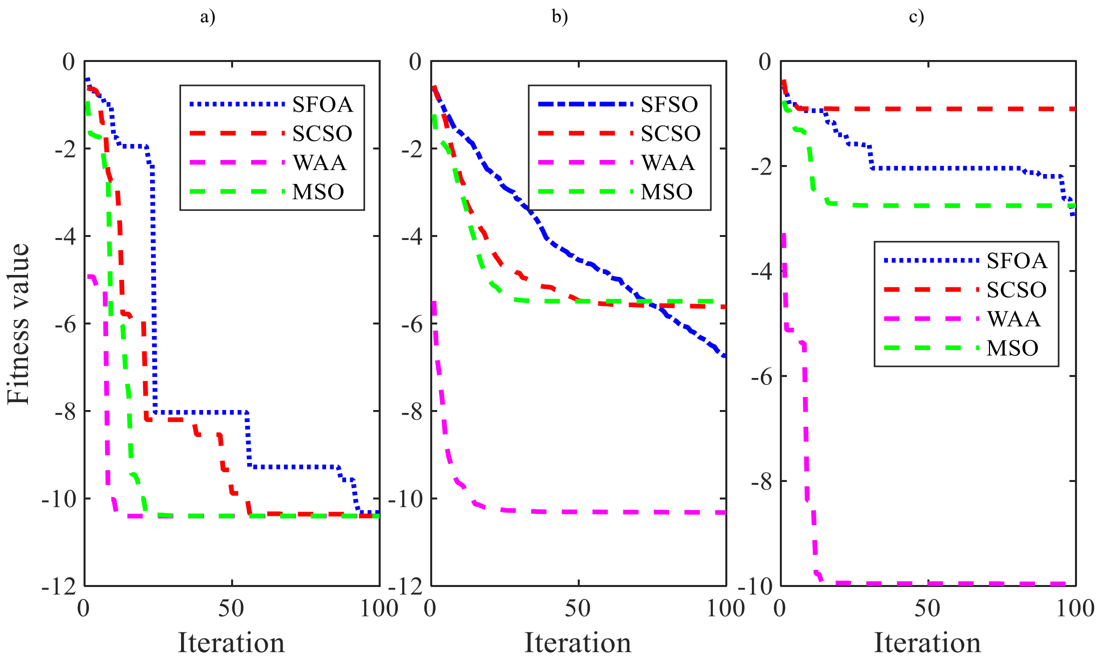

Figure 8: a) The minimum, b) mean, and c) maximum convergence obtained by the four applied algorithms for their best runs while dealing with the fourth optimization problem.

Table 4: The summarized results of the four algorithms dealing with the fourth optimization problem.

	SFOA	SCSO	WAA	MSO
Min.Fit	-10.320	-10.40	-10.403	-10.403
Ave.Fit	-6.737	-5.616	-10.320	-5.489
Max.Fit	-2.973	-0.908	-9.961	-2.752
Std	2.301	3.188	0.110	3.194

with zero fluctuation and consistently achieving the global minimum (near -10.3). In the fifth problem's 50-run test, SFOA, SCSO, and MSO once again exhibit extreme, wide-ranging volatility. This massive, unpredictable fluctuation is equally pronounced and damaging as that observed in the fourth problem, confirming that the high complexity and multimodal nature of these functions severely compromise the search mechanisms of SFOA, SCSO, and MSO, rendering them unsuitable for robust, consistent optimization. Furthermore, the presentation of Figure 9, followed by Figure 8, which presents the same information, is another indication of the struggle of SFOA, SCSO, and MSO to reach the optimal value of the main objective function as the overall complexity of the given optimization problem increases. For WAA, on the other hand, the presence of Figure 9 is another demonstration of its unmatched search performance. These claims are further enhanced when observing Figure 10, which shows the performance of the four algorithms in their best runs. Clearly, WAA is completely superior to others in terms of the convergence speed to the optimal fitness values. As in the previous optimization problems, the qualitative results obtained by the four applied algorithms are shown in Table 5. The results in the table consistently confirm the clear superiority of WAA over the other in all comparison criteria, especially the Min.Fit and the value of Std. Clearly, WAA has proven itself to be a powerful and stable optimization tool across the five optimization problems.

5. Conclusions

In this study, the search performance of the four applied algorithms, including the Starfish

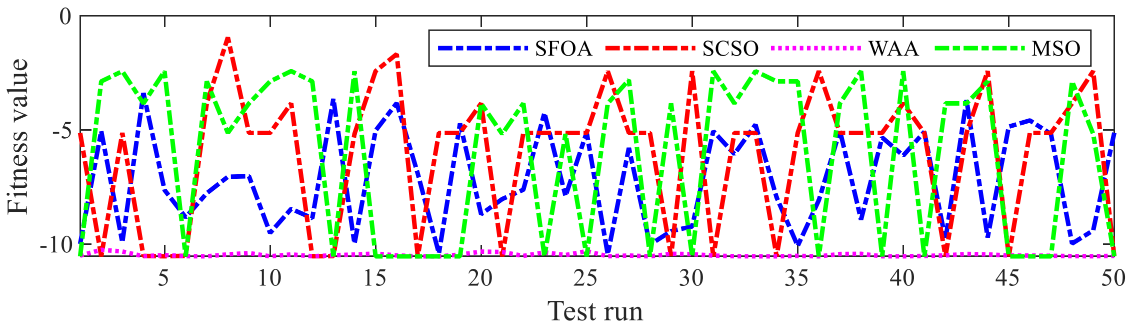

Figure 9: The fitness values obtained by the four applied algorithms after 50 test run the fifth optimization problem.

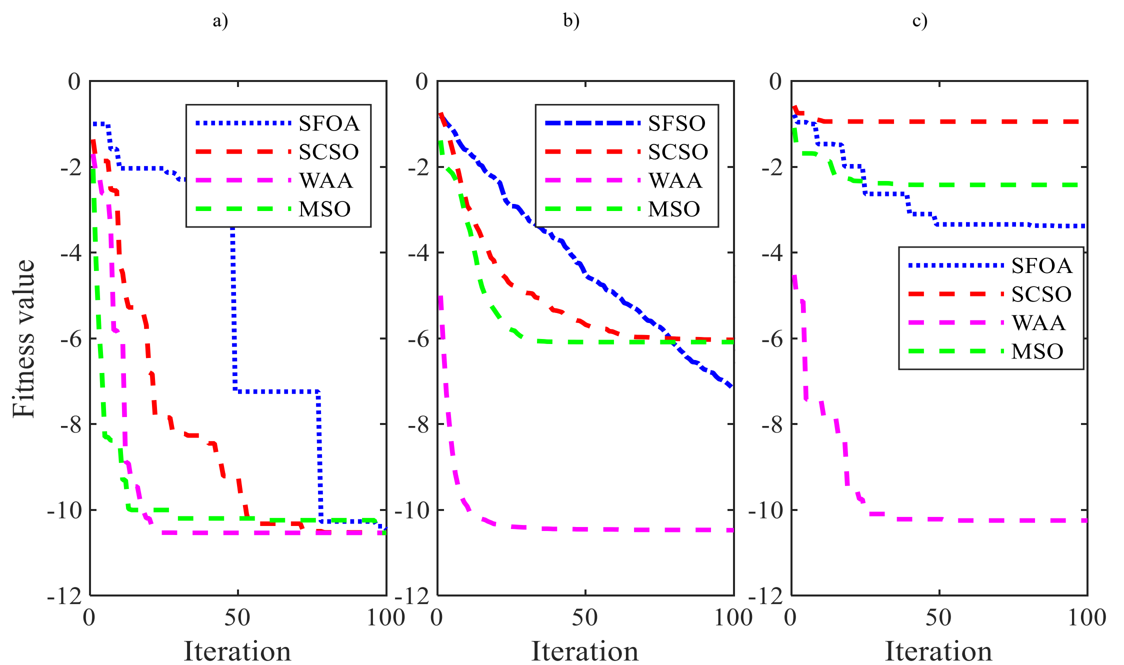

Figure 10: a) The minimum, b) mean, and c) maximum convergence obtained by the four applied algorithms for their best runs while dealing with the fifth optimization problem.

Table 5: The summarized results of the four algorithms dealing with the fifth optimization problem.

	SFOA	SCSO	WAA	MSO
Min.Fit	-10.451	-10.535	-10.536	-10.536
Ave.Fit	-7.185	-6.034	-10.474	-6.087
Max.Fit	-3.381	-0.946	-10.249	-2.422
Std	2.230	3.027	0.071	3.589

optimization algorithm, the Sand Cat swarm optimization, the Weighted Average Algorithm, and the Mirage search optimization, is investigated in detail on five test functions using different criteria. The four applied algorithms are compared to each other on the fluctuation among 50 test runs, and three convergence characteristics, including the minimum, the mean, and the maximum convergence, are considered throughout the five test functions. The results indicate that WAA is the best applied algorithm among the four in terms of providing and maintaining stability search performance with smaller fluctuations and fast response time, represented by the number of iterations needed to reach the optimal values of the considered optimization problems. Based on these results and performance, WAA is considered a powerful and robust search method; therefore, it is strongly recommended for use in dealing with such optimization problems.

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