Single Phase Second Order Sliding Mode Controller for Mismatched Uncertain Systems with Extended Disturbances and Unknown Time-Varying Delays

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(Received: 10-Oct-2021; accepted: 29-Jan-2022; published: 30-Jun-2022)
DOI: http://dx.doi.org/10.55579/jaec.202262.353

Abstract. *In this paper, a novel single phase second order sliding mode controller (SP-SOSMC) is proposed for the mismatched uncertain systems with extended disturbances and unknown time-varying delays. The main achievements of this study consist of three tasks: 1) a reaching phase in conventional sliding mode control (CSMC) technique is removed to ensure the global stability of the system; 2) an influence of the undesired high-frequency oscillation phenomenon in control input is vanished; 3) an exogenous perturbation is generally extended to the k-order disturbance of state variable. Firstly, a single phase switching manifold function is defined to eliminate the reaching phase in CSMC. Secondly, an unmeasurable state variable is estimated by using the proposed reduced-order sliding mode observer (ROSMO) tool. Next, a SPSOSMC is built based on the help of ROSMO tool and output information only. Then, a sufficient condition is established by employing the linear matrix inequality (LMI) technique and Lyapunov function theory such that the resulting sliding mode dynamics is asymptotically stable. Finally, a numerical example is simulated via the well-known MATLAB software to validate the effectiveness of the proposed technique.*

Keywords

Second order sliding mode control, chattering removal, exogenous disturbance, without reaching phase.

1. Introduction

Sliding mode control (SMC) technique that originated from the theory of variable structure control (VSC) has attracted noticeable amount of interest. The remarkable features include robustness property, rapid convergence, simplicity of implementation, and insensitivity to plant parameter variations and uncertainties [1, 2]. In virtue of these advantages, SMC has been successfully applied for practical control systems comprising mechanical systems, solar photovoltaic energy systems, wind turbines, induction motor drives, nuclear reactor, etc. [3]-[5].

Unfortunately, since the controller switches between two structures during performance process, the plant will suffer high-frequency oscillation near the switching surface. This negative phenomenon, so-called "chattering", is caused by the discontinuous control signal. The chattering has many serious impacts in practical control applications since it may damage the control actuator, reduce the control accuracy, and excite the unwanted unmodeled dynamics [6, 7]. Among the various techniques to solving this disadvantage, the concept of second order sliding mode control methodology was first developed in the 1980s by [8, 9]. The key idea of this method is the discontinuous sign function added to the control law's time derivative and thus real control input signal after integration is continuous which suppresses the chattering [10]. For example, a second order sliding mode controller (SOSMC) was proposed in [11] for reducing the chattering and stabilizing a single-input, single-output (SISO) nonlinear systems with incomplete state availability. By using adaptive tuning law in the proposed controller, a prior knowledge about the upper bound of the system uncertainties is eliminated for a class of dynamic system [12]. In [13], a chattering-free second-order fast terminal sliding mode control law is designed based on a linear matrix inequality (LMI) for a class of non-linear uncertain systems with mismatched uncertainties. Nevertheless, these studies did not consider the external perturbations which effect on the plant. In [14], a full-order observer (FOO)-based SOSMC was established for stabilizing a linear multi-input, multi-output (MIMO) uncertain system with matched uncertainties. A super-twisting control signal was proposed based on super-twisting observer for perturbed double integrator systems [15]. By using a back stepping-like method, a novel SOSMC controller was explored in [16] for nonlinear system with mismatched term. The controller design problem was investigated based on Lyapunov analysis for second order sliding mode dynamics subject to mismatched unbounded perturbation [17]. Recently, a novel SOSMC was designed based on linear quadratic regulator in [18] for mismatched uncertain nonlinear fractional-order systems. In [19], by building a new Barrier Lyapunov Function and employing the adding a power integrator technique, a novel second-order sliding mode control algorithm was designed to handle the output constraint problem. Also, based on this technique, a new SOSMC was synthesized for nonlinear systems with mismatched term and time-varying output constraint [20]. In [21], a new SOSMC was proposed for the nonlinear system by using the backstepping-like method and virtual control strategy. Based on the Lyapunov method, an adaptive SOSMC was designed in [22] for nonlinear systems with uncertainties. However, the authors in the researches [18]-[22] assumed that the state variables of the system are available. This is not feasible in practice due to expensive sensor costs or some unmeasurable variables. In addition, the external disturbance and the time delays in the systems are not considered, which is important in both theory and real-world applications. Furthermore, in the existing publications of the VSC, motion dynamics is determined after the state trajectories of system reach the switching surface and the performance of plant is unknown in the reaching phase. Consequently, the system's whole stability may not be guaranteed or seriously corrupted [23, 24]. To solve this drawback, integral SMC (ISMC) technique has been developed [25]-[27]. In [25], a self-tuning higher order sliding mode controller was proposed for a class of SISO nonlinear uncertain systems. Following this technique, a second-order integral sliding mode control law was proposed in [26] for a class of uncertain systems with input time delay. Compared to the traditional SMC, the ISMC method have three key advantages [13]. First, the ISMC removes the reaching phase so that the stability of the whole system can be ensured. Second, the ISMC is sensitive to external disturbance and parameters uncertainty. Third, the response of the system with the matched disturbance is identical to the response of the nominal plant. The control signal of the ISMC technique includes two main terms, the first term is the continuous nominal control which is used for controlling the nominal plant and the second term is discontinuous control that is employed to robust against the effects of exogenous perturbation. However, due to the existence of the discontinuous function, the controller offers high-frequency chattering phenomenon. Thus, it is essential for control systems to improve a novel single phase second order sliding mode control law removing the reaching phase and reducing the undesired chattering phenomenon.

Motivated by all the previous works and the mentioned limitations above, we will address a new single phase second order sliding mode controller (SPSOSMC) based on a new reduced-order sliding mode observer (ROSMO) for the mismatched uncertain systems with unknown time-varying delays and extended perturbations. The main contributions include three key aspects:

- A new switching manifold fuction is defined to eliminate the reaching phase in CSMC and the robustness performance against exogenous perturbations is exactly ensured from the initial time instance.
- A SPSOSMC is constructed based on a new ROSMO with lower dimension to settle the chattering phenomenon in control input and stabilize for mismatched uncertain time-varying delay systems.
- A external disturbance is generally extended to the order disturbance of state variable which the major condition set by the recent studies [18, 26] (that the exogenous perturbations must be bounded by a known function or constant) is mitigated.

The organization of this study is planned as follows. Section 2 gives a brief description of the problem and the establishment of the regular form. In Section 3, the key contributions which include the asymptotic stability of closed-loop system in sliding mode, construction of a novel ROSMO and the design of a SPSOSMC via ROSMO tool are presented. The simulation example that discussed in Section 4 is displayed to demonstrate the effectiveness of the proposed technique. Finally, the concluding remarks and suggestions for further goal are drawn in Section 5.

2. Model description of the system

Consider a class of the mismatched uncertain time-delay systems with extended disturbances whose dynamics are represented by the following state-space form as:

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]\, x(t) + [A_d + \Delta A_d(t)]\, x_d(t) \\ \quad + B\,[u(t) + \zeta(x(t), x_d, t)]\,, \\ y(t) = Cx(t), \\ x_d = x(t - d(t)) \text{ and } x(t) = \chi(t), \text{ for } t \in [-\bar{d}, 0) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state position vector, $u(t) \in R^m$ is the control force, and $y(t) \in R^p$ is the output vector. The matrices A, A_d, B and C are constant matrices with proper dimensions. The terms $\Delta A(t)$ and $\Delta A_d(t)$ are mismatched uncertainties parameters in the state matrix and the delayed state matrix of the plant, respectively. The symbol $\zeta(x(t), x_d, t)$ is a matched non-linearity of the plant. The time-varying delay $d(t)$ is assumed to be unknown function, nonnegative, and bounded in $\Re^+$; that is, $\bar{d} := \sup_{t\in\Re^+} \{d(t)\} < \infty$. The sign $\chi(t)$ denotes a continuous initial function with differentiable vector on $[-\bar{d}, 0]$.

In order to complete the description of the mismatched uncertain time-delay systems (1), the following standard assumptions is valid:

Assumption 1. The number of output channels is greater than or equal to the number of control inputs and is smaller than the number of the system's state variables, that is, $m \le p \le n$. The matrices B and C have full column rank, and $\mathrm{rank}(CB) = \mathrm{rank}(B) = m$.

Assumption 2. The pair (A, B) and (A, C) are entirely controllable and observable, respectively.

Assumption 3. The mismatched uncertainties parameters $\Delta A(t)$ and $\Delta A_d(t)$ satisfy the following form

$$\begin{aligned} &[\Delta A(t)\Delta A_d(t)] \\ &\quad = [D\Sigma(x(t), t)ED_d\Sigma_d(x(t), x_d, t)E_d]\,, \end{aligned} \tag{2}$$

where D, E, D_d and E_d are non-unique known constant matrices with appropriate dimensions. Further, the matrices $\Sigma(x(t), t)$

and $\Sigma_d(x(t), x_d, t)$ are unknown functions but bounded by $\|\Sigma(x(t), t)\| \leq 1$ and $\|\Sigma_d(x(t), x_d, t)\| \leq 1$ for all $(x(t), t) \in R^n \times R$ and $(x(t), x_d, t) \in R^n \times R$, respectively.

To achieve a regular form of the original systems (1) which will develop a new controller later, the obtained results of the paper [28] will be used in this paper. We definite Γ an $n \times n$ symmetric matrix gratifying $\Gamma = I - E^g E$, where I is an identity matrix, $B^{\perp}$ is any matrix whose columns form the basis of the null space of the matrix B^T and E^g is the Moore-Penrose inverse of the matrix E. That is, we only investigate a general problem case satisfying the mismatching condition. To get a new chattering-free single phase output feedback controller, a single phase switching manifold function is defined as:

$$s(y(t), t) = \dot{\sigma}(y(t), t) + \bar{R}\sigma(y(t), t), \tag{3}$$

where $\sigma(y(t), t) = \bar{\sigma}(y, t) - \bar{\sigma}(y, 0)\exp(-\alpha t)$, $\dot{\sigma}(y(t), t)$ is time derivative of the function $\sigma(y(t), t)$, $\bar{\sigma}(y, t) = Sx = Fy$, $\bar{R} \in R^{m\times m}$ is any diagonal matrix, and α is positive scalar. Besides, S is sliding matrix and F is selected matrix such that the equation $S = FC$ is solvable. The matrix S can be expressed as $S = KB^T(\Gamma P\Gamma + BQB^T)^{-1} = KB^T T^{-1}$, where $K \in R^{m\times m}$ is any non-singular matrix and P, Q are symmetric matrices that will be answered in the solutions of the following LMIs:

$$\begin{cases} \Gamma P\Gamma + BQB^T > 0, \\ B^{\perp T}\left(A\Gamma P\Gamma + \Gamma Q\Gamma \mathrm{A}^{\mathrm{T}}\right)B^{\perp} < 0. \end{cases} \tag{4}$$

Remark 1. *By extending the concept without reaching phase of papers [23, 29], the single phase switching manifold is proposed in Eq. (3). When $\sigma(y(0), 0) = 0$, reaching time is equal to zero and the sliding mode exists from beginning time. Therefore, the robustness of system is ensured during whole intervals of control action.*

Remark 2. *Compared with the previous ISMC technique [25]-[27], a novel SOSMC design with the single phase sliding function (3) can be easily implemented and the undesired high-frequency oscillation phenomenon in control input can be completely reduced.*

Now, we use a new transformation matrix that will transform the original systems (1) into a regular form. The following state transformation can be defined as

$$[\vartheta(t)\sigma(t)]^T = Mx \text{ and } [\vartheta_d(t)\sigma_d(t)]^T = Mx_d \tag{5}$$

Where $M = [B^{\perp T} KB^T(\Gamma P\Gamma + BQB^T)^{-1}]^T$, and its inverse matrix $M^{-1} = [TB^{\perp}(B^{\perp T}TB^{\perp})^{-1}B(SB)^{-1}]$. By substituting state transformation (5) into (1), we can get

$$\begin{cases} \dot{\vartheta}(t) = \left[\bar{A}_{11} + \Delta\bar{A}_{11}\right]\vartheta(t) + \left[\bar{A}_{12} + \Delta\bar{A}_{12}\right]\sigma(t) \\ +\left[\bar{A}_{11d} + \Delta\bar{A}_{11d}\right]\vartheta_d(t) + \left[\bar{A}_{12d} + \Delta\bar{A}_{12d}\right]\sigma_d(t) \\ \dot{\sigma}(t) = \left[\bar{A}_{21} + \Delta\bar{A}_{21}\right]\vartheta(t) + \left[\bar{A}_{22} + \Delta\bar{A}_{22}\right]\sigma(t) \\ +\left[\bar{A}_{21d} + \Delta\bar{A}_{21d}\right]\vartheta_d(t) + \left[\bar{A}_{22d} + \Delta\bar{A}_{22d}\right]\sigma_d(t) \\ +(SB)\left[u(t) + \zeta(x(t), x_d, t)\right], \end{cases} \tag{6}$$

where

$$\bar{A}_{11} + \Delta\bar{A}_{11}(t) = B^{\perp T}\left[A + D\Sigma(x,t)E\right]TB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$$

$$\bar{A}_{12} + \Delta\bar{A}_{12} = B^{\perp T}\left[A + D\Sigma(x,t)E\right]B(SB)^{-1}$$

$$\bar{A}_{21} + \Delta\bar{A}_{21} = KB^T T^{-1}\left[A + D\Sigma(x,t)E\right]TB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$$

$$\bar{A}_{22} + \Delta\bar{A}_{22} = KB^T T^{-1}\left[A + D\Sigma(x,t)E\right]B(SB)^{-1}$$

$$\bar{A}_{11d} + \Delta\bar{A}_{11d} = B^{\perp T}\left[A_d + D_d\Sigma_d(x(t), x_d, t)E_d\right]TB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$$

$$\bar{A}_{12d} + \Delta\bar{A}_{12d} = B^{\perp T}\left[A_d + D_d\Sigma_d(x(t), x_d, t)E_d\right]B(SB)^{-1}$$

$$\bar{A}_{21d} + \Delta\bar{A}_{21d} = KB^T T^{-1}\left[A_d + D_d\Sigma_d(x(t), x_d, t)E_d\right]TB^{\perp}[B^{\perp T}TB^{\perp}]^{-1}$$

$$\bar{A}_{22d} + \Delta\bar{A}_{22d}(t) = RB^T T^{-1}\left[A_d + D_d\Sigma_d(x(t), x_d, t)E_d\right]B(SB)^{-1}$$

$$\vartheta = B^{\perp T}x, \sigma = Fy - Fy(y,0)\exp(-\alpha t)$$

$$\vartheta_d = B^{\perp T}x_d$$

$$\sigma_d = Fy_d - Fy_d(y,0)\exp(-\alpha t).$$

According to the obtained results in the published paper [28], we have

$$\begin{aligned} \Delta\bar{A}_{11} &= B^{\perp T}D\Sigma ETB^{\perp}(B^{\perp T}TB^{\perp})^{-1} = 0, \\ \Delta\bar{A}_{21} &= KB^T T^{-1}D\Sigma ETB^{\perp}(B^{\perp T}TB^{\perp})^{-1} = 0, \\ \Delta\bar{A}_{11d} &= B^{\perp T}D_d\Sigma_d E_d TB^{\perp}(B^{\perp T}TB^{\perp})^{-1} = 0, \\ \Delta\bar{A}_{21d} &= KB^T T^{-1}D_d\Sigma_d E_d TB^{\perp}(B^{\perp T}TB^{\perp})^{-1} = 0. \end{aligned} \tag{7}$$

The regular form (6) can be represented as

$$\begin{cases}\dot{\vartheta}(t)=\bar{A}_{11}\vartheta(t)+\left[\bar{A}_{12}+\Delta\bar{A}_{12}(t)\right]\sigma(t)+\bar{A}_{11d}\vartheta_d(t)\\+\left[\bar{A}_{12d}+\Delta\bar{A}_{12d}(t)\right]\sigma_d(t),\\\dot{\sigma}(t)=\bar{A}_{21}\vartheta(t)+\left[\bar{A}_{22}+\Delta\bar{A}_{22}(t)\right]\sigma(t)+\bar{A}_{21d}\vartheta_d(t)\\+\left[\bar{A}_{22d}+\Delta\bar{A}_{22d}\right]\sigma_d+(SB)\left[u(t)+\zeta(x,x_d,t)\right].\end{cases}\tag{8}$$

3. Main results

3.1. Establishment of a novel ROSMO

In this section, we will propose a novel ROSMO which helps to design a SPSOSMC for the time-varying delay systems (1). We design a following ROSMO that will estimate the unmeasurable variables of the plant.

$$\dot{\hat{\vartheta}}(t)=\bar{A}_{11}\hat{\vartheta}(t)+\bar{A}_{12}\sigma(t)+\bar{A}_{11d}\hat{\vartheta}_d(t)+\bar{A}_{12d}\sigma_d(t)\tag{9}$$

where $\hat{\vartheta}(t)$ and $\hat{\vartheta}_d(t)$ are the estimate of $\vartheta(t)$ and $\vartheta_d(t)$, respectively. And $\hat{\vartheta}(t)=\hat{\phi}_{\hat{\vartheta}}(t)=B^{\perp T}\hat{\phi}(t)$ with $t\in[-\bar{d},0]$. Let us denote an error difference between the estimate variables and the real variables as $\tilde{\vartheta}(t)=\hat{\vartheta}(t)-\vartheta(t)$ and $\tilde{\vartheta}_d(t)=\hat{\vartheta}_d(t)-\vartheta_d(t)$. Then, combining the first Eq. (6), the results (7), and the Eq. (9) lead to the observer error dynamics as

$$\begin{aligned}\dot{\tilde{\vartheta}}(t)&=B^{\perp T}ATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\tilde{\vartheta}\\&+B^{\perp T}A_dTB^{\perp}\times(B^{\perp T}TB^{\perp})^{-1}\tilde{\vartheta}_d\\&-B^{\perp T}D\Sigma(x,t)EB(SB)^{-1}\sigma\\&-B^{\perp T}D_d\Sigma_d(x(t),x_d,t)E_dB(SB)^{-1}\sigma_d.\end{aligned}\tag{10}$$

Remark 3. *In the first equation of the regular form (8), the state variables and are unmeasurable. In order to estimate these unmeasurable states, the new ROSMO (9) is designed. Based on the achieved results in the paper [30], the matrices $\bar{A}_{11}$ and $\bar{A}_{11d}$ are stable. Consequently, the ROSMO (9) and the observer error dynamics (10) are asymptotically stable in sliding mode, $\sigma(t)=\sigma_d(t)=0$. In other words, the estimated variables $\hat{\vartheta}(t)$ and $\hat{\vartheta}_d(t)$ tend to the original variables $\vartheta(t)$ and $\vartheta_d(t)$, respectively. Therefore, the unmeasurable states $\vartheta(t)$ and $\vartheta_d(t)$ are estimated by the proposed observer (9).*

To calculate the upper bound of observer error dynamics, we will propose a novel theorem as below

Theorem 1. *Let $\tilde{\vartheta}(0)$ be an initial condition of the observer error dynamics $\tilde{\vartheta}(t)$. The norm of $\left\|\tilde{\vartheta}(t)\right\|$ is bounded by $\tilde{\varpi}(t)$, which is result of*

$$\begin{aligned}\dot{\tilde{\varpi}}(t)&=\delta\tilde{\varpi}(t)+\tilde{\varepsilon}\tilde{\eta}_1[\left\|B^{\perp T}D\right\|\left\|EB(SB)^{-1}\right\|\\&+\tilde{\eta}_2\left\|B^{\perp T}D_d\right\|\left\|E_dB(SB)^{-1}\right\|]\left\|\sigma(t)\right\|,\end{aligned}\tag{11}$$

where $\delta=\lambda_{\max}+\tilde{\varepsilon}\tilde{\eta}_1\left\|B^{\perp T}A_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|<0$, $\lambda_{\max}$ is maximum eigenvalue of the matrix $B^{\perp T}ATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$, and $\tilde{\varepsilon}>0$, $\tilde{\eta}_1>1$, $\tilde{\eta}_2>1$. The initial value of the error upper bound $\tilde{\varpi}(0)\geq\tilde{\varepsilon}\left\|\tilde{\vartheta}(0)\right\|>0$.

Proof. Based on the proved results in [30], the matrices $B^{\perp T}ATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$ and $B^{\perp T}A_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$ are stable. Hence, we get $\left\|\exp(B^{\perp T}ATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}t)\right\|\leq\tilde{\varepsilon}\exp(\lambda_{\max}t)$, where $\tilde{\varepsilon}$ is positive scalar. By solving (10) to yields

$$\begin{aligned}&\left\|\tilde{\vartheta}(t)\right\|\leq\left\|\exp\left(B^{\perp T}ATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}t\right)\right\|\left\|\tilde{\vartheta}(0)\right\|\\&+\int_0^t\left\|\exp\left[B^{\perp T}AQB^{\perp}(B^{\perp T}QB^{\perp})^{-1}(t-\tau)\right]\right\|\\&\times\left[\left\|B^{\perp T}A_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|\left\|\tilde{\vartheta}_d(\tau)\right\|\right.\\&+\left\|B^{\perp T}D\Sigma(x,t)EB(SB)^{-1}\right\|\left\|\sigma(\tau)\right\|\\&\left.+\left\|B^{\perp T}D_d\Sigma_dE_dB(SB)^{-1}\right\|\left\|\sigma_d(\tau)\right\|\right]d\tau,\end{aligned}\tag{12}$$

By applying the Lemma 3 of the paper [31], we have $\left\|\tilde{\vartheta}_d(t)\right\|\leq\tilde{\eta}_1\left\|\tilde{\vartheta}(t)\right\|$ and $\left\|\sigma_d(t)\right\|\leq\tilde{\eta}_2\left\|\sigma(t)\right\|$ for some scalars $\tilde{\eta}_1>1$ and $\tilde{\eta}_2>1$, respectively. The inequality (12) can be rewritten as

$$\begin{aligned}&\left\|\tilde{\vartheta}(t)\right\|\leq\tilde{\varepsilon}\left\|\tilde{\vartheta}(0)\right\|\exp\left(\lambda_{\max}t\right)+\int_0^t\tilde{\varepsilon}\exp\left[\lambda_{\max}(t-\tau)\right]\\&\times\left[\tilde{\eta}_1\left\|B^{\perp T}A_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|\left\|\tilde{\vartheta}(\tau)\right\|\right.\\&+\left(\left\|B^{\perp T}D\Sigma EB(SB)^{-1}\right\|+\tilde{\eta}_2\left\|B^{\perp T}D_d\Sigma_d\right.\right.\\&\left.\left.\times E_dB(SB)^{-1}\right\|\right)\left\|\sigma(\tau)\right\|\left.\right]d\tau.\end{aligned}\tag{13}$$

Let us multiply the term $\exp(-\lambda_{\max}t)$ both sides of the inequality (13), we achieve

$$\begin{aligned}&\left\|\tilde{\vartheta}(t)\right\|\exp\left(-\lambda_{\max}t\right)\le\tilde{\varepsilon}\left\|\tilde{\vartheta}(0)\right\|+\\&\int_0^t\tilde{\varepsilon}\exp\left[\lambda_{\max}(t-\tau)\right]\\&\times\left[\tilde{\eta}_1\left\|B^{\perp T}A_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\right\|\left\|\tilde{\vartheta}(\tau)\right\|\right.\\&+\left(\left\|B^{\perp T}D\right\|\left\|EB(SB)^{-1}\right\|+\tilde{\eta}_2\left\|B^{\perp T}D_d\right\|\right.\\&\left.\left.\times\left\|E_dB(SB)^{-1}\right\|\right)\left\|\sigma(\tau)\right\|\right]d\tau.\end{aligned}\tag{14}$$

Shift $\exp(-\lambda_{\max}t)$ to the right-hand side term of inequality (14) and utilize the Lemma of the paper [32], it follows that

$$\begin{aligned}&\left\|\tilde{\vartheta}(t)\right\|\le\tilde{\varpi}(0)\exp\\&\left[\left(\lambda_{\max}+\tilde{\varepsilon}\tilde{\eta}_1\left\|B^{\perp T}A_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\right\|\right)t\right]\\&+\int_0^t\tilde{\varepsilon}\tilde{\eta}_1\exp\\&\left[\begin{array}{l}\left(\lambda_{\max}+\tilde{\varepsilon}\tilde{\eta}_1\left\|B^{\perp T}A_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\right\|\right)\\\times(t-\tau)]\\\left[\left(\left\|B^{\perp T}D\right\|\left\|EB(SB)^{-1}\right\|+\tilde{\eta}_2\left\|B^{\perp T}D_d\right\|\right.\right.\\\left.\left.\times\left\|E_dB(SB)^{-1}\right\|\right)\left\|\sigma(\tau)\right\|\right]d\tau=\tilde{\varpi}(t)\end{array}\right],\end{aligned}\tag{15}$$

where $\tilde{\varpi}(t)$ satisfies (15). Hence, we can conclude that $\left\|\tilde{\vartheta}(t)\right\|\le\tilde{\varpi}(t)$ for all time. Thus, the proof is finished. □

Now, we are in position to derive sufficient conditions by LMI such that the closed-loop system is asymptotically stable in the sliding mode.

3.2. Asymptotically stable condition by LMI

In this section, a well-known LMI technique, the Lyapunov theory, and some properties of the time delay variable will be used to demonstrate the asymptotical stability of the closed-loop system in the sliding mode. Let us consider the following LMI such that the first Eq. (6) is an asymptotically stable.

$$\begin{bmatrix}\Omega & \hat{E}^T & \hat{E}_d^T & \hat{\mathrm{X}}\hat{D} & \hat{\mathrm{X}}\hat{D}_d\\\hat{E} & -\mathit{\Gamma}^{-1}I & 0 & 0 & 0\\\hat{E}_d & 0 & -(\beta_2\mathit{\Gamma}_{1d})^{-1}I & 0 & 0\\\hat{D}^T\hat{\mathrm{X}} & 0 & 0 & -\mathit{\Gamma}^{-1}I & 0\\\hat{D}_d^T\hat{\mathrm{X}} & 0 & 0 & 0 & -\mathit{\Gamma}_{1d}^{-1}I\end{bmatrix}<0\tag{16}$$

where $\Omega=\hat{A}^T\hat{\mathrm{X}}+\hat{\mathrm{X}}\hat{A}+\mathit{\Gamma}_{2d}^{-1}\hat{\mathrm{X}}+\beta_1\mathit{\Gamma}_{2d}\hat{A}_d^T\hat{\mathrm{X}}\hat{A}_d$, $\hat{A}=B^{\perp T}\times ATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$, $\hat{A}_d=B^{\perp T}A_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$, $\hat{D}=B^{\perp T}D$, $\hat{D}_d=B^{\perp T}D_d$, $\hat{E}=ETB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$, $\hat{E}_d=E_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}$, the scalars $\beta_1>1,\beta_2>1$, the signs $\mathit{\Gamma},\mathit{\Gamma}_{1d}$, and $\mathit{\Gamma}_{2d}$ are positive constants, and $\hat{\mathrm{X}}\in R^{(n-m)\times(n-m)}$ is any positive definite matrix. Then, we can build the following Theorem 2.

Theorem 2. *Suggest that the LMI (16) has a feasible solution* $\hat{\mathrm{X}}>0$, *the scalars* $\beta_1>1,\beta_2>1$, *the signs* $\mathit{\Gamma}>0,\mathit{\Gamma}_{1d}>0$, *and* $\mathit{\Gamma}_{2d}>0$. *The specified switching manifold surface is designed as the Eq. (3). Then, the first Eq. (6) of the mismatched uncertain time-delay systems is asymptotically stable in sliding mode.*

Proof. In the sliding mode, we get $\sigma(y(t),t)=0$. The first Eq. (6) is represented as

$$\begin{aligned}\dot{\vartheta}(t)=&\left[\hat{A}+\hat{D}\Sigma(x,t)\hat{E}\right]\vartheta(t)\\&+\left[\hat{A}_d+\hat{D}_d\Sigma_d(x(t),x_d,t)\hat{E}_d\right]\vartheta_d(t),\end{aligned}\tag{17}$$

where the matrices $\hat{A},\hat{D},\hat{E},\hat{A}_d,\hat{D}_d$, and $\hat{E}_d$ are constant matrices which is defined in (16).

Now, we consider the candidate Lyapunov functional $V(t)=\vartheta^T\hat{\mathrm{X}}\vartheta$, where $\hat{\mathrm{X}}\in R^{(n-m)\times(n-m)}$ is positive definite matrix. Calculating the time derivative of $V(t)$ and employing the Eq. (17) yield

$$\begin{aligned}&\dot{V}(t)=\vartheta^T(t)\{\hat{A}^T\hat{\mathrm{X}}+\hat{\mathrm{X}}\hat{A}+\hat{E}^T\Sigma^T(x,t)\\&\times\hat{D}^T\hat{\mathrm{X}}+\hat{\mathrm{X}}\hat{D}\Sigma(x,t)\hat{E}\}\vartheta(t)\\&+\vartheta^T{}_d(t)\hat{E}_d^T\Sigma_d^T(x(t),x_d,t)\hat{D}_d^T\hat{\mathrm{X}}\vartheta(t)\\&+\vartheta^T(t)\hat{\mathrm{X}}\hat{D}_d\Sigma_d(x(t),x_d,t)\hat{E}_d\vartheta_d(t)\\&+\vartheta^T{}_d(t)\hat{A}_d^T\hat{\mathrm{X}}\vartheta(t)+\vartheta^T(t)\hat{\mathrm{X}}\hat{A}_d\vartheta_d(t).\end{aligned}\tag{18}$$

We are going to verify $\dot{V}(t)<0$. By applying the Lemma 1 of paper [33], it follows from the Eq. (18) that

$$\begin{aligned}&\dot{V}(t)\le\vartheta^T\\&\left\{\hat{A}^T\hat{\mathrm{X}}+\hat{\mathrm{X}}\hat{A}+\mathit{\Gamma}^{-1}\hat{\mathrm{X}}\hat{D}\hat{D}^T\hat{\mathrm{X}}+\mathit{\Gamma}\hat{E}^T\hat{E}\right\}\vartheta\\&+\mathit{\Gamma}_{1d}^1\vartheta^T\hat{\mathrm{X}}\hat{D}_d\hat{D}_d^T\hat{\mathrm{X}}\vartheta+\mathit{\Gamma}_{1d}\vartheta^T{}_d\hat{E}_d^T\hat{E}_d\vartheta_d\\&+\vartheta^T\hat{\mathrm{X}}\hat{A}_d\vartheta_d+\vartheta^T{}_d\hat{A}_d^T\hat{\mathrm{X}}\vartheta,\end{aligned}\tag{19}$$

where $\mathit{\Gamma}$ and $\mathit{\Gamma}_{1d}$ are positive scalars.

Based on the Assumption 3, The matrix E is selected to obtain the semi-positive definite matrix E^TE. Then, using the Lemma 3 of [31] and the Lemma 2 of [34], the inequality (19) is equivalent to

$$\dot{V}(t) \le \vartheta^T(t)\left\{\Omega + \Gamma^{-1}\hat{\mathrm{X}}\hat{D}\hat{D}^T\hat{\mathrm{X}} + \Gamma\hat{E}^T\hat{E} + \Gamma_{1d}^{1}\hat{\mathrm{X}}\hat{D}_d\hat{D}_d^T\hat{\mathrm{X}} + \beta_2\Gamma_{1d}\hat{E}_d^T\hat{E}_d\right\}\vartheta(t), \tag{20}$$

where $\Omega = \hat{A}^T\hat{\mathrm{X}} + \hat{\mathrm{X}}\hat{A} + \Gamma_{2d}^{-1}\hat{\mathrm{X}} + \beta_1\Gamma_{2d}\hat{A}_d^T\hat{\mathrm{X}}\hat{A}_d$.

Next, applying the Schur complement formula in paper [35] to LMI (16), we have

$$\Omega + \Gamma^{-1}\hat{\mathrm{X}}\hat{D}\hat{D}^T\hat{\mathrm{X}} + \Gamma\hat{E}^T\hat{E} + \Gamma_{1d}^{1}\hat{\mathrm{X}}\hat{D}_d\hat{D}_d^T\hat{\mathrm{X}} + \beta_2\Gamma_{1d}\hat{E}_d^T\hat{E}_d < 0. \tag{21}$$

Combining the inequalities (20) and (21), we can see that $\dot{V}(t) < 0$, which further implies that the sliding motion (17) is asymptotically stable. The proof of Theorem 2 is ended. □

To continue a new chattering-free controller design which uses second order SMC technique, we will present it in next section.

3.3. Design of a chattering-free SOSMC

In this section, we will design a SPSOSMC based on the output information and the estimated variables from the ROSMO (9) such that the plant states are driven into the switching manifold in finite time and remain on it thereafter. Firstly, the time derivative of the switching function $\sigma(y(t), t)$ is given by

$$\begin{aligned}&\dot{\sigma}(y(t), t) = \\ &SATB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\vartheta(t) + SAB(SB)^{-1}\sigma(t) \\ &+ SA_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\vartheta_d(t) \\ &+SA_dB(SB)^{-1}\sigma_d(t) \\ &+ (SB)u(t) + \psi(t) + \alpha\bar{\sigma}(y,0)\exp(-\alpha t),\end{aligned} \tag{22}$$

where

$$\begin{aligned}\psi(t) &= S\Delta A(t)TB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\vartheta(t) \\ &+ S\Delta A_d(t)TB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\vartheta_d(t) \\ &+ S\Delta A(t)B(SB)^{-1}\sigma(t) + S\Delta A_d(t) \\ &\times B(SB)^{-1}\sigma_d(t) + (SB)\zeta(x(t), x_d, t).\end{aligned} \tag{23}$$

The key idea of the second order sliding mode is to perform the second order derivative of the switching variable $\ddot{\sigma}(y(t), t)$ rather than the first derivative as in traditional sliding mode. The second order derivative with respect time for the switching variable is found as

$$\begin{aligned}&\ddot{\sigma}(y(t), t) = \\ &SATB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\dot{\vartheta}(t) + SAB(SB)^{-1}\dot{\sigma}(t) \\ &+ SA_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\dot{\vartheta}_d(t) \\ &+SA_dB(SB)^{-1}\dot{\sigma}_d(t) \\ &+ (SB)\dot{u}(t) + \dot{\psi}(t) - \alpha^2\bar{\sigma}(y,0)\exp(-\alpha t).\end{aligned} \tag{24}$$

The sliding manifold and its derivative are respectively expressed as

$$\begin{aligned}s(t) &= SATB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\vartheta(t) \\ &+SAB(SB)^{-1}\sigma(t) \\ &+ SA_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\vartheta_d + SA_dB(SB)^{-1}\sigma_d \\ &+ (SB)u(t) + \psi(t) + \alpha\bar{\sigma}(y,0)\exp(-\alpha t) \\ &+ \bar{R}\left[\bar{\sigma}(t) - \bar{\sigma}(y,0)\exp(-\alpha t)\right],\end{aligned} \tag{25}$$

and

$$\begin{aligned}\dot{s}(t) &= SATB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\dot{\vartheta} \\ &+SAB(SB)^{-1}\dot{\sigma} \\ &+ SA_dTB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\dot{\vartheta}_d + SA_dB(SB)^{-1}\dot{\sigma}_d \\ &+ (SB)\dot{u}(t) + \dot{\psi}(t) - \alpha^2\bar{\sigma}(y,0)\exp(-\alpha t) \\ &+ \bar{R}\left[\dot{\sigma}(y,t) + \alpha\bar{\sigma}(y,0)\exp(-\alpha t)\right].\end{aligned} \tag{26}$$

Secondly, an unknown external disturbance $\dot{\psi}(t)$ in the Eq. (26) is assumed to be met the following condition

$$\left\|\dot{\psi}(t)\right\| \le \sum_{k=0}^{k}\left[\upsilon_k(\|x\|)^k\right], \tag{27}$$

where k is the disturbance order and υ_k is unknown unknown positive constant corresponding to $k-$order. For example, if the order of perturbation is 2, then $\left\|\dot{\psi}(t)\right\| \le \upsilon_0 + \upsilon_1(\|x\|) + \upsilon_2(\|x\|)^2$. However, in practical control systems, $k-$order and constant υ_k are unknown in advance due to the complexity of system structure and are determined by the designer. By replacing (5) into (27) and using $\|\vartheta\| \le \left\|\hat{\vartheta}(t)\right\| + \tilde{\varpi}(t)$, the inequality (27) can be represented as

$$\left\|\dot{\psi}(t)\right\| \le \sum_{k=0}^{k}\left[\upsilon_k\left(\begin{array}{c}\left\|TB^{\perp}\left(B^{\perp T}TB^{\perp}\right)^{-1}\right\| \\ \left(\left\|\hat{\vartheta}(t)\right\| + \tilde{\varpi}(t)\right) + \\ \left\|B(SB)^{-1}\right\|\|\sigma\|\end{array}\right)^k\right] \tag{28}$$

Now, a chattering-free SOSMC is constructed based on the achieved results in Theorems 1 and 2 for reducing the destructive high-frequency oscillatory in control input and stabilizing the mismatched uncertain time-varying delay systems (1). This is key contribution of this study.

To force the state variables of the varying-time delay systems (8) on the specified switching manifold (3) in finite time and stay on thereafter, a new output feedback SOSMC is proposed as

$$\begin{gathered}\dot{u}(t) = -(SB)^{-1}\left\{\tilde{\rho}_1\left[\left\|\hat{\vartheta}\right\| + \tilde{\varpi}\right] + \tilde{\rho}_2\left\|\sigma\right\| \right.\\ + \tilde{\rho}_3\left\|\dot{\sigma}\right\| + \tilde{\rho}_4 + \tilde{\alpha}\left\|s(t)\right\| + \left\|\bar{R}\right\|\left\|\bar{\sigma}(y,t)\right\| \\ \left. + \left[\bar{R}\alpha - \alpha^2\right]\bar{\sigma}(y,0)\exp(-\alpha t)\right\} sign\left(s(t)\right),\end{gathered} \tag{29}$$

where $\tilde{\alpha}$ is positive scalar, $\tilde{\rho}_1, \tilde{\rho}_2, \tilde{\rho}_3$,and $\tilde{\rho}_4$ are control gains that will determine later.

The overall block diagram of the proposed scheme in Fig. 1 includes three main blocks, the controlled system (1), ROSMO (9) and the controller (29). The control input contains two parts, the variables estimated by ROSMO and measured output signal $y(t)$.

Theorem 3. *Suppose that the LMI (16) has a solution* $\hat{\mathrm{X}}> 0$*, and the scalars* $\beta_1 > 1, \beta_2 > 1$*, the signs* $\Gamma> 0, \Gamma_{1d}> 0$*, and* $\Gamma_{2d}> 0$*. Consider the mismatched uncertain time-delay systems (1) subject to Assumptions 1-3. If the switching manifold function (3), the ROSMO (9), and the chattering-free SOSMC (29) are employed and the observer error dynamics (10) satisfies the Theorem 1, then the state trajectories of the plant are driven into the switching manifold surface* $s(y(t),t) = 0$ *in finite time and stay on the sliding mode under the control signal (29) when scalar gains gratify the following settings*

$$\begin{gathered}\tilde{\rho}_1 \geq \left\|SATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|\left[\left\|\bar{A}_{11}\right\| + \tilde{\eta}_3\left\|\bar{A}_{11d}\right\|\right] \\ + \tilde{\eta}_2\left\|SA_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|\left\|\bar{A}_{11}\right\| \\ + \tilde{\eta}_2\tilde{\eta}_3\left\|SA_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|\left\|\bar{A}_{11d}\right\|, \\ \tilde{\rho}_2 \geq \left\|SATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\right\|\left[\left\|\bar{A}_{12}\right\| + \left\|B^{\perp T}D\right\| \right.\\ \times\left\|EB(SB)^{-1}\right\| + \left\|\bar{A}_{12d}\right\| + \tilde{\eta}_2\left\|B^{\perp T}D_d\right\| \\ \left.\times\left\|E_dB(SB)^{-1}\right\|\right] + \tilde{\eta}_2\left\|SA_dTB^{\perp}\ (B^{\perp T}TB^{\perp})^{-1}\right\| \\ \times\left[\left\|\bar{A}_{12}\right\| + \left\|B^{\perp T}D\right\|\left\|EB(SB)^{-1}\right\| \right.\\ \left. + \left\|\bar{A}_{12d}\right\| + \tilde{\eta}_2\left\|B^{\perp T}D_d\right\|\left\|E_dB(SB)^{-1}\right\|\right], \\ \tilde{\rho}_3 \geq \left\|SAB(SB)^{-1}\right\| + \tilde{\eta}_3\left\|SA_dB(SB)^{-1}\right\|, \\ \tilde{\rho}_4 \geq \left\|\dot{\psi}(t)\right\|.\end{gathered} \tag{30}$$

Proof. Consider the candidate Lyapunov functional as $V(s) = \|s(y(t),t)\|$, where direct differentiation of $V(s)$ results

$$\begin{gathered}\dot{V}(s) = \tfrac{s^T(t)}{\|s(t)\|}\{SATB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\dot{\vartheta} \\ +SAB(SB)^{-1}\dot{\sigma} \\ + SA_dTB^{\perp}(B^{\perp T}TB^{\perp})^{-1}\dot{\vartheta}_d + SA_dB(SB)^{-1}\dot{\sigma}_d \\ + (SB)\dot{u}(t) + \dot{\psi}(t) + \bar{R}[\bar{\sigma}(y,t) + \alpha\bar{\sigma}(y,0) \\ \times\exp(-\alpha t)] - \alpha^2\bar{\sigma}(y,0)\exp(-\alpha t)\},\end{gathered} \tag{31}$$

By applying the Lemma 3 of the paper [31], we have $\|\sigma_d(t)\| \leq \tilde{\eta}_2\|\sigma(t)\|$ and $\|\vartheta_d(t)\| \leq \tilde{\eta}_3\|\vartheta(t)\|$. In addition, $\|\vartheta(t)\| \leq \left\|\hat{\vartheta}(t)\right\| + \tilde{\varpi}(t)$. By combining the first Eq. (8) and Eq. (31), we have

$$\begin{gathered}\dot{V}(s) \leq \tilde{\rho}_1[\left\|\hat{\vartheta}(t)\right\| + \tilde{\varpi}(t)] + \tilde{\rho}_2\|\sigma(t)\| + \tilde{\rho}_3\|\dot{\sigma}(t)\| \\ + \tilde{\rho}_4 + [\bar{R}\alpha - \alpha^2]\bar{\sigma}(y,0)\exp(-\alpha t) \\ + s^T(t)/\|s(t)\|(SB)\dot{u}(t) + \left\|\bar{R}\right\|\|\bar{\sigma}(y,t)\|,\end{gathered} \tag{32}$$

where the control gains $\tilde{\rho}_1, \tilde{\rho}_2, \tilde{\rho}_3$, and $\tilde{\rho}_4$ are specified in inequality (30).

Now, by replacing the control signal (29) into (32), it is clear that $\dot{V}(s) \leq -\tilde{\alpha}\|s(t)\|$, where $\tilde{\alpha}$ is a positive scalar. Hence, the state trajectories of the mismatched uncertain time-delay systems (1) hit the switching surface manifold $s(y(t),t) = 0$ in finite time and stay on it. The proof of Theorem 3 is completed. □

Remark 4. *From single phase switching manifold (3) and Theorems 2 and 3, we can see that there are only the estimated variables and output information utilized in the controller (29). Unlike existing studies [25, 26], this work is no need to require that the plant states are available.*

Remark 5. *Compared with recent publications that used ISMC technique [25]-[27], the proposed method has improved features such as the desired dynamic behaviour of the plant are attained from the beginning time* $t \geq 0$*, the stability of the whole system can be guaranteed for all time, and the influence of chattering in input control is eliminated.*

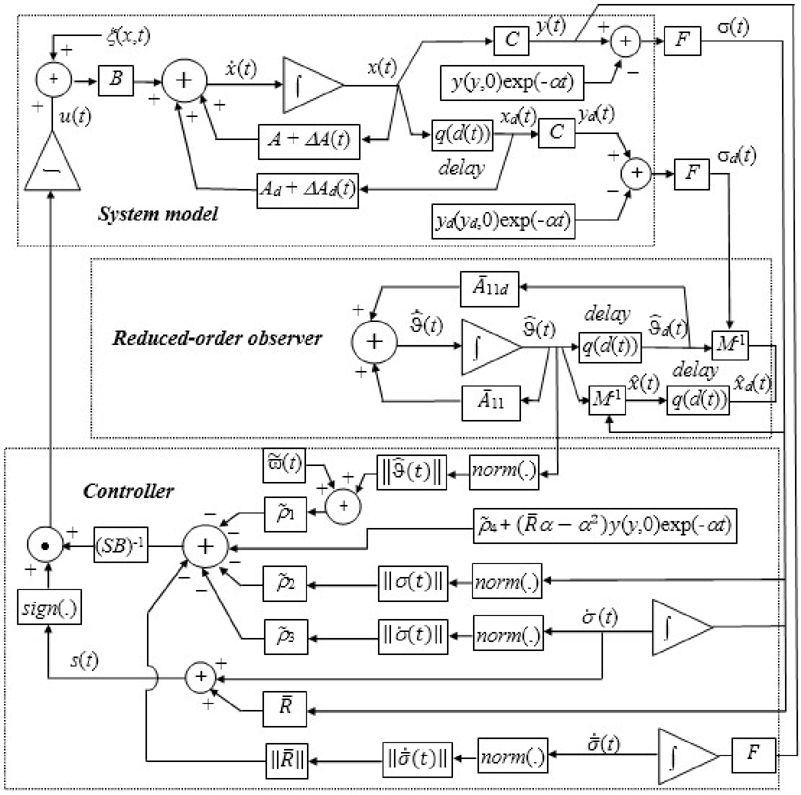

Fig. 1: Block diagram of the system

4. Illustrated example

In this section, the above proposed results will be illustrated by the numerical simulation that modified from the paper [36]. The mathematical model of the mismatched uncertain time-varying delay systems is described as

$$\dot{x}(t) = \left\{ \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ -1 & 1 & -0.75 \end{bmatrix} + D\Sigma(x(t),t)E \right\} x(t)$$
$$+ \left\{ \begin{bmatrix} 1 & 0.5 & -0.5 \\ -0.5 & 0 & 0.5 \\ 1 & 1 & -0.5 \end{bmatrix} + D_d \Sigma_d(x(t), x_d, t) E_d \right\}$$
$$\times x_d(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [u(t) + \zeta(x(t), x_d, t)],$$
$$y(t) = Cx(t) = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} x(t), \tag{33}$$

where $x(t) = [x_1(t)x_2(t)x_3(t)]^T \in R^3$, $u(t) \in R^1$, $y(t) = [y_1(t)y_2(t)]^T \in R^2$. The parameter mismatched uncertainties in state matrix and delayed state matrix are respectively $D\Sigma(x(t),t)E = [001]^T \Sigma(x(t),t)\,[110]$ and $D_d\Sigma_d(x(t),x_d,t)E_d = [010]^T \Sigma_d(x(t),x_d,t)\,[110]$ with $\Sigma(x(t),t) = 0.14\sin(0.1t)$ and $\Sigma_d(x(t),x_d,t) = 0.22\sin(0.1t)$. The external perturbation is assumed to be 2-order and satisfied the following inequality.

$$\left\| \dot{\psi}(t) \right\| \leq 0.011 + 0.021\left(\|x\|\right) + 0.0182(\|x\|)^2. \tag{34}$$

In this simulation, the setting parameters are given as: $\alpha = 0.47, \tilde{\alpha} = 0.33, \eta_2 = 0.01$, $\tilde{\varepsilon}= 0.011$, $\tilde{\eta}_1= 1.33$, $\tilde{\eta}_2= 1.72$, $\tilde{\eta}_3= 1.58$, $\beta_1 = \beta_2 = 1.1$, $\varGamma=\varGamma_{1d}=\varGamma_{2d}= 0.1$. The initial conditions of the state variables, the output, the observer error dynamics, and the error upper bound are $x(0) = [0.1 - 0.10.2]^T$, $y(0) = [1 - 1]^T$, $\tilde{\vartheta}(0) = [0.30.1]^T$, and $\tilde{\varpi}(0) = 0.0035$,, respectively. For goal of simulation, let the unknown time-varying delay be $d(t) = 0.15(1 + \sin 0.5t)$ [37]. Following this problem, we have $\mathrm{rank}[B\Delta A] = 2 > \mathrm{rank}[B] = 1$. In order words, the time-varying delay system (33) does not need to fulfill the so-called matching condition.

By solving the LMIs (4) via MATLAB's LMI Control Toolbox, we find a feasible answer as follows

$$\begin{aligned} P &= \begin{bmatrix} 103.821 & 43.365 & -6.563 \\ 43.365 & -15.887 & -6.224 \\ -6.563 & -6.224 & 1.276 \end{bmatrix}, \\ Q &= [0.9509]. \end{aligned} \tag{35}$$

Next, we calculate the sliding matrix F by solving the equation $FC = KB^T(\Gamma P\Gamma + BQB^T)^{-1}$, and chose the matrices $\bar{R} = K = [1]$. From the Eq. (25), the sliding manifold is computed as

$$\begin{aligned} s(y(t), t) &= \begin{bmatrix} 1.0516 & 1.0516 \end{bmatrix} \dot{y} \\ &+ \begin{bmatrix} 1.0516 & 1.0516 \end{bmatrix} y \\ &+ \begin{bmatrix} 0.4943 & 0.4943 \end{bmatrix} y(0) \exp(-0.47t), \end{aligned} \tag{36}$$

Following the Eq. (9), the new ROSMO is given by

$$\begin{aligned} \dot{\hat{\vartheta}}(t) &= \begin{bmatrix} -2.0000 & -0.0000 \\ 2.0000 & -0.7500 \end{bmatrix} \hat{\vartheta} + \\ &\begin{bmatrix} -0.9509 \\ 0.9509 \end{bmatrix} \sigma \\ &+ \begin{bmatrix} 0.5000 & 0.5000 \\ -0.0000 & -0.5000 \end{bmatrix} + \begin{bmatrix} -0.4755 \\ 0.9509 \end{bmatrix} \sigma_d, \end{aligned} \tag{37}$$

where $\sigma = \begin{bmatrix} 1.0516 & 1.0516 \end{bmatrix} y - \begin{bmatrix} 0.2620 & 0.2620 \end{bmatrix} y(0) \times \exp(-0.47t)$ and $\sigma_d = \begin{bmatrix} 1.0516 & 1.0516 \end{bmatrix} y_d - \begin{bmatrix} 0.2620 & 0.2620 \end{bmatrix} \times y_d(0) \exp(-0.47t)$. Acting the Eq. (10), the time-varying delay observer error dynamics is found as

$$\begin{aligned} \dot{\tilde{\vartheta}}(t) &= \begin{bmatrix} -2.00 & -0.000 \\ 2.00 & -0.750 \end{bmatrix} \tilde{\vartheta} \\ &+ \begin{bmatrix} 0.500 & 0.500 \\ -0.000 & -0.5000 \end{bmatrix} \tilde{\vartheta}_d \\ &- \begin{bmatrix} 0 & 1 \end{bmatrix} T \sum(x, t) \begin{bmatrix} 1.0000 & 1.0000 \end{bmatrix} y \\ &- \begin{bmatrix} 0 & 1 \end{bmatrix} T \sum(x(t), x_d, t) \begin{bmatrix} 1.0000 & 1.0000 \end{bmatrix} y_d, \end{aligned} \tag{38}$$

where $\Sigma(x(t), t) = 0.14 \sin(0.1t)$ and $\Sigma_d(x(t), x_d, t) = 0.22 \sin(0.1t)$. By solving LMI (16), it is easy to verify that condition in Theorem 2 is satisfied with positive Theorem 2 is satisfied with positive matrix $\hat{\mathrm{X}} = \begin{bmatrix} 5.6480 & 2.6537 \\ 2.6537 & 4.0268 \end{bmatrix}$. Then, a SPSOSMC is constructed by

$$\begin{aligned} u(t) = &-0.9509 \int_0^t \Big\{ 12.9040 \left[\left\| \hat{\vartheta}(t) \right\| + \tilde{\varpi}(t) \right] \\ &+ 13.1704 \left\| \sigma \right\| + 2.0005 \left\| \dot{\sigma} \right\| + 0.33 \left\| s(t) \right\| \\ &+ 1.4872 \left\| \dot{y} \right\| + \tilde{\rho}_4 + \begin{bmatrix} 0.2620 & 0.2620 \end{bmatrix} y(0) \\ &\times \exp(-0.47t) \Big\} \, sign\left(s(t)\right) dt, \end{aligned} \tag{39}$$

where $s(t)$ is switching manifold specified in (36), $\hat{\vartheta}(t)$ is solution of the ROSMO (37), the upper bound of the estimate error $\tilde{\varpi}(t)$ is an answer of $\dot{\tilde{\varpi}} = -0.7382\tilde{\varpi}(t) + 2.8284e^{-08} \left\| y \right\|$, and $\tilde{\rho}_4 = 0.011 + 0.0297[\left\| \hat{\vartheta}(t) \right\| + \tilde{\varpi}(t)] + 0.0297 \left\| y \right\| + 0.0182[1.4142(\left\| \hat{\vartheta}(t) \right\| + \tilde{\varpi}(t)) + 1.4142 \left\| y \right\|]^2$. The simulation results are respectively depicted from Fig. 2 to Fig. 7 including the state variables of the plant, the switching manifold, the ROSMO, the time-varying delay observer error dynamics, the upper bound of the error dynamics, and the new SPSOSMC.

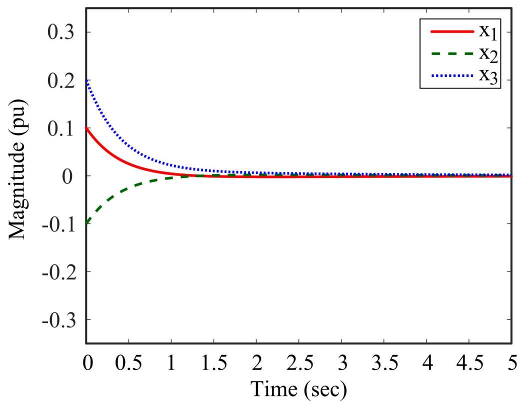

Fig. 2: Time response of the plant states.

Remark 6. *The graphical representations of system state variables are plotted in Fig. 2 under the proposed SPSOSMC (39). From this figure, it can be clearly seen that the system states decline immediately to zero after about 1.8 seconds. In other words, when the proposed SPSOSMC is employed, the state trajectories of the system approach to the manifold surface $s(y(t), t) = 0$ in short time where the published researches [19, 25] could not be replicated the attainments.*

Remark 7. *The time history of the switching manifold surface $s(y(t), t)$ (36) is shown in Fig. 3. It can be seen that $s(y(t), t)$ tends toward to*

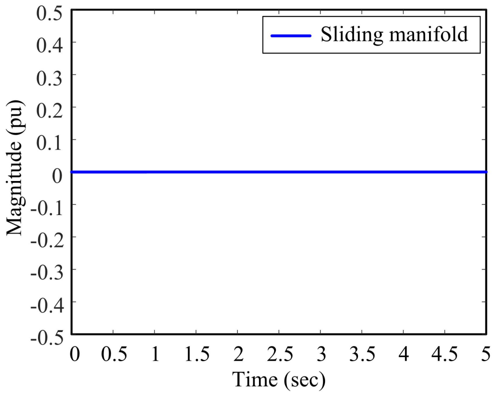

Fig. 3: Time history of the switching manifold.

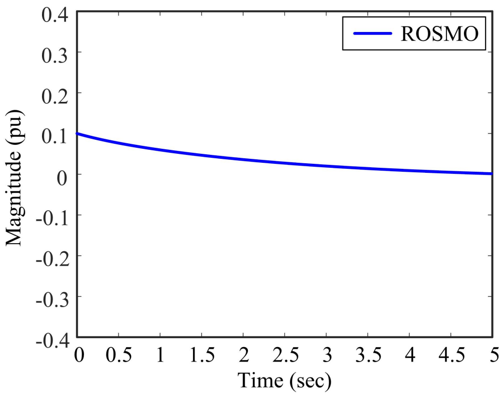

Fig. 4: Time response of the observer.

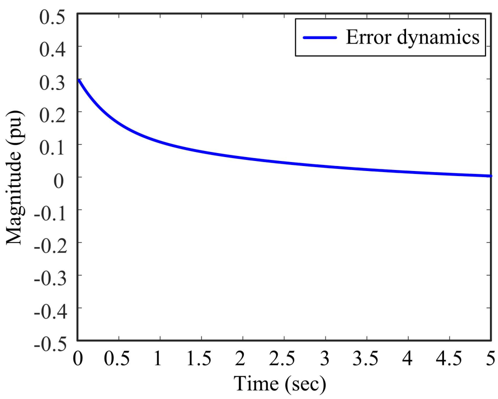

Fig. 5: The trajectory of the error dynamics.

zero from the initial time instance and remain on the specified switching manifold surface. Obviously, it can see that the plant is more robust against disturbances than the traditional SMC technique [17]-[19].

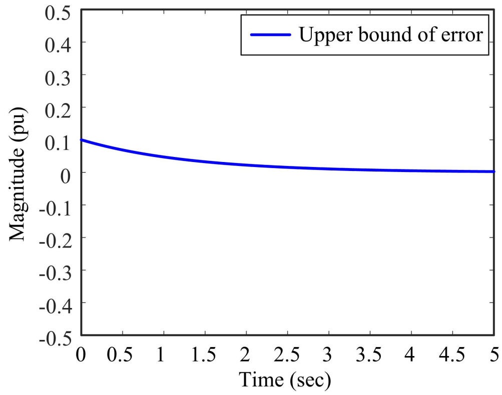

Fig. 6: The trajectory of the error's upper bound.

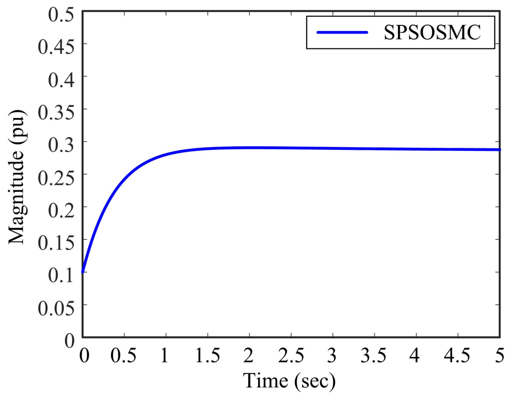

Fig. 7: Time response of the proposed controller.

Remark 8. *The time evolution of the reduced-order estimator and its error dynamics are respectively depicted in Fig. 4 and Fig. 5. From these figures, we can see that the estimation error (38) is exactly driven into zero by the designed ROSMO (37). The time response of the error dynamics converges to zero after about 3.8 seconds. This result verifies that the estimated variable $\hat{\vartheta}(t)$ which displayed in Fig. 4 tends toward to the real variable of the plant $\vartheta(t)$. In addition, the observer error is bounded by the upper bound $\tilde{\varpi}(t)$ (11), whose dynamics response is exhibited Fig. 6. Thus, the proposed ROSMO with lower dimension ensures that the conservatism is decreased, and the robustness of the system is increased in comparison with FOOs [14, 38].*

Remark 9. *The time history of the control signal $u(t)$ (39) is exposed in Fig. 7. From this figure, it is noted that the response of the plant is obtained with the suggested SPSOSMC (39)* *guaranteeing faster response and quite smaller amplitude. In the proposed controller, we can see that there are only measurable output and the estimated variables used. Unlike the suggested controllers in the published works [25, 26], the state information is assumed to be available, which are not designed for this problem. In addition, when the sign function is added in the proposed second order SMC law (39), the chattering in the control input is completely addressed.*

From the aforementioned exploration of the achieved results, it is concluded that the proposed technique is effective in dealing with the reaching phase and chattering suppression problems of the mismatched uncertain delay systems with extended disturbances and unknown time-varying delays.

5. Conclusion

This paper has presented a new single phase second order sliding mode controller (SPSOSMC) for the mismatched uncertain systems. The system includes the unknown time-varying delays in advance and the extended disturbances which are extended to $k-$order disturbances of the state variables. The SPSOSMC has been established based on the estimated variables from ROSMO tool and output information only. With this controller, the system's robustness has been enhanced, conservatism has been reduced, and the high-frequency oscillation phenomenon has been eliminated. Moreover, the sufficient condition in terms of LMIs is given to ensure the sliding mode asymptotical stability of the closed-loop system. Finally, the achieved simulation results demonstrate the theoretical analysis and exhibit that the suggested technique effectively controls a mismatched uncertain time-varying systems with extended disturbances. Hence, the application of the proposed method to the practical control systems such as electrical drives, electrical power systems, mobile robots, and spacecraft in continuous time domain could be the future trend.

Acknowledgement

The authors wish to express their gratitude to Van Lang University, Vietnam for financial support for this research.

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